Appendix B

Thornthwaite - Mather

The Thornthwaite - Mather equation for estimating potential evapotranspiration was developed in the 1940s and 1950s by Thornthwaite and Mather at the Laboratory of Climatology in Centerton, NJ. They made measurements of evapotranspiration around the world and noticed that potential evapotranspiration appeared to be a function of the average temperature, the latitude of the measurement, and the length of the day. They developed an empirical equation that uses monthly average temperature as a measure of the energy available for evapotranspiration. Although they felt that the method was not mathematically fully developed, they considered it a good approximation for potential evapotranspiration. The method is still widely used. The equation has no correction for different vegetation types and is commonly stated as:

$$pET = .63[50(T-32)/(9I)]^{a} \text{ x lcf}$$

Where: pET = potential evapotranspiration (in/month)
T = average monthly temperature (°F)
I = annual thermal index
a = calculated constant
lcf = latitude correction factor

For each month, a monthly heat index (i) is calculated as a function of the average monthly temperature (T): $i = ((T - 32)/9)^{1.514}$. The monthly heat indices are summed to give an annual thermal index (I). "a" is calculated as a function of the annual thermal index (I):

$$a = (6.75 \times 10^{-7})I^3 - (7.71 \times 10^{-5})I^2 + (1.79 \times 10^{-2})I + .49239$$

Finally, the latitude correction factor (lcf) is based on the number of daylight hours per month which is a function of the solar declination and the latitude of the study site:

lcf = (daylight hours for entire month) / (12 * number of days in the month)
daylight hours per day = (24 / π) * (Arccos(-Tan(latitude) * Tan(solar declination)))
latitude (in radians) = (2π * (latitude (in degrees))) / 360
solar declination (in radians) = 0.4093 * Sin(((2πJ) / 365) - 1.405)
J = Julian day number

The empirical equation holds for temperatures between 32° and 79.7° F. If the temperature is less than 32° F, the potential evapotranspiration is zero. If the temperature is greater than 79.7° F, then potential evapotranspiration is given by:

$$PET = lcf (((-5.25625072726565E-03 * T^2) + (1.04170341298537 * T) - 44.3259754866234)).$$

Turc

The Turc equation is an empirical radiation-based equation for calculating potential evapotranspiration. The method has been shown to perform well in humid climates. The equation requires estimates or measurements of net solar radiation (Sn), average daily temperature (T), and daily relative humidity (RH):

If the relative humidity is less than 50%, then:

$$pET = (0.313 / 25.4) (T / (T + 15)) (Sn + 2.1) (1 + ((50 - RH) / 70))$$

If the relative humidity is greater than or equal to 50%, then:

$$pET = (0.313 / 25.4) (T / (T + 15)) (Sn + 2.1)$$

where: pET = potential evapotranspiration (inches/day)
T = daily average temperature (C)
RH = daily relative humidity (%)
Sn = daily net solar radiation (mm/day)

Our model assumes that solar radiation data are not available and calculates the daily net solar radiation as a function of the daily extraterrestrial radiation (So), the surface albedo (α), and the number of daily measured sunshine hours (n/N):

$$Sn = (So) (1 - \alpha) (a_s +(b_s)(n/N))$$

Where: So = extraterrestrial radiation ($MJ/m^2 day$)
α = albedo = .23 is recommended in absence of knowledge of land cover
a_s = 0.25 for average climates
b_s = 0.50 for average climates
n = bright sunshine hours per day (hours)
N = total day length (hours)

The daily extraterrestrial radiation (So) is calculated as a function of the sunset hour angle (ω_s), the latitude (ϕ), and the solar declination (δ):

$$So = 15.392\, d_r\, (\omega_s\ Sin(\phi)\ Sin(\delta) + Cos(\phi)\ Cos(\delta)\ Sin(\omega_s))$$

Where: So = daily extraterrestrial radiation (mm/day)
d_r = relative distance between the earth and sun
ω_s = sunset hour angle (radians)
ϕ = latitude (radians)
δ = solar declination (radians)

The relative distance between the earth and sun (d_r) is given by:

$$d_r = 1 + (0.033 (\text{Cos}(2\pi J) / 365))$$

Where: J = Julian day number

The sunset hour angle (ω_s) is given by:

$$\omega_s = \text{Arccos}(- \text{Tan}(\phi) \text{ Tan}(\delta))$$

The solar declination (δ) is given by:

$$\delta = 0.4093 \text{ Sin}((2\pi J) / 365) - 1.405)$$

Where: J = Julian day number

To convert the daily extraterrestrial radiation from mm/day to MJ/m^2day, multiply So by the density of water (0.001 g/mm^3) and the latent heat of vaporization (2447.22 J/g) and convert to meters.

The total day length (N) can be calculated from:

$$N = (24 / \pi) \ \omega_s$$

Like the Thornthwaite equation, if the temperature is less than 32° F, the potential evapotranspiration is assumed to be zero; otherwise, the equation holds.

<u>Jensen-Haise</u>

The Jensen - Haise equation is an empirical radiation-based equation for calculating potential evapotranspiration. The equation requires estimates or measurements of net solar radiation (Sn) and average daily temperature (T):

$$pET = (0.41 / 25.4) \text{ Sn } ((0.025 \ T) + 0.078)$$

where: pET = potential evapotranspiration (inches/day)
T = daily average temperature (°C)
Sn = daily net solar radiation (mm/day)

As with the Turc method described above, the model assumes that solar radiation data are not available and calculates the daily net solar radiation as a function of the daily extraterrestrial radiation (So), the surface albedo (α), and the number of daily measured sunshine hours (n/N):

$$Sn = So (1 - \alpha) \ (a_s + b_s \ (n/N))$$

Where: So = extraterrestrial radiation (MJ/m^2day)
α = albedo = .23 is recommended in absence of knowledge of land cover
a_s = 0.25 for average climates
b_s = 0.50 for average climates

n = bright sunshine hours per day (hours)
N = total day length (hours)

The daily extraterrestrial radiation (So) is calculated as a function of the sunset hour angle (ω_s), the latitude (ϕ), and the solar declination (δ):

$$So = 15.392\ d_r\ (\omega_s\ \text{Sin}(\phi)\ \text{Sin}(\delta) + \text{Cos}(\phi)\ \text{Cos}(\delta)\ \text{Sin}(\omega_s))$$

Where: So = daily extraterrestrial radiation (mm/day)
d_r = relative distance between the earth and sun
ω_s = sunset hour angle (radians)
ϕ = latitude (radians)
δ = solar declination (radians)

The relative distance between the earth and sun (d_r) is given by:

$$d_r = 1 + (0.033\ (\text{Cos}(2\pi J) / 365))$$

Where: J = Julian day number

The sunset hour angle (ω_s) is given by:

$$\omega_s = \text{Arccos}(-\ \text{Tan}(\phi)\ \text{Tan}(\delta))$$

The solar declination (δ) is given by:

$$\delta = 0.4093\ \text{Sin}((2\pi J) / 365) - 1.405)$$

Where: J = Julian day number

To convert the daily extraterrestrial radiation from mm/day to MJ/m^2day, multiply So by the density of water (0.001 g/mm^3) and the latent heat of vaporization (2447.22 J/g) and convert to meters.

The total day length (N) can be calculated from:

$$N = (24 / \pi)\ \omega_s$$

Like the Thornthwaite equation, if the temperature is less than 32° F, the potential evapotranspiration is assumed to be zero; otherwise, the equation holds.

Blaney-Criddle Equation

The Blaney – Criddle equation is primarily a temperature-based approach for estimating potential evapotranspiration. The equation was originally developed in 1950 and has been widely applied for irrigation designs in the western United States. The method is still in common use and is stated in its most modern complex form as:

$$E = a_{BC} + b_{BC} f$$

With: $f = p(.46T + 8.13)$
$a_{BC} = 0.0043RH_{min} - (n/N) - 1.41$
$b_{BC} = 0.81917 - 0.0040922(RH_{min}) + 1.0705(n/N) + 0.065649(U_d)$
$- 0.0059684(RH_{min})(n/N) - 0.0006967(RH_{min})(U_d)$

where: p = ratio of monthly daytime hours to annual daytime hours (%)
T = average monthly air temperature (°C)
(n/N) = ratio of the actual to possible monthly sunshine hours
RH_{min} = average minimum monthly relative humidity (%)
U_d = average monthly daytime wind speed at 2 meters height (m/sec)

The total day length (N) can be calculated from:

$$N = (24 / \pi)\ \omega_s$$

Where: ω_s = sunset hour angle (in radians)

The sunset hour angle (ω_s) is given by:

$$\omega_s = \text{Arccos}(-\ \text{Tan}(\phi)\ \ \text{Tan}(\delta))$$

Where: ϕ = latitude (in radians)
δ = solar declination (in radians)

The solar declination (δ) is given by:

$$\delta = 0.4093\ \ \text{Sin}((2\pi J) / 365) - 1.405)$$

Where: J = Julian day number

Like the Thornthwaite equation, if the temperature is less than 32° F, the potential evapotranspiration is assumed to be zero; otherwise, the equation holds.