COLOR MIXTURE IN COMPUTER GRAPHICS

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Abstract
Digital control of color television monitors has added precise control of a large subset of human colorspace to the capabilities of computer graphics. This subset is the set of colors spanned by the red, green, and blue electron guns exciting their respective phosphors. A color can be represented as a triple of numbers between zero and one, representing the excitement levels of the respective guns. This capability allows the creation of new and the testing of old mathematical formulae regarding color mixture.

This paper presents the three basic models of color mixture (additive, subtractive, and pigmentary), as well as algorithms for computation of color resulting from mixture of arbitrary amounts of two colors under either of the three methods. Guidelines for extension of the algorithms to deal with simultaneous mixture of more than two colors are provided. Particular emphasis is placed on pigmentary mixture, with a discussion of a new geometric model, in which the hexagon is presented as a shape more consistent for modeling the pigmentary color gamut than the canonical circle.

INTRODUCTION
The human visual system analyzes color according to the levels of three primary components (red, green, and blue). Color television monitors thus span human colorspace by varying the amounts of red, green, and blue \((rgb)\) phosphor excitement. Since computer graphics programs normally interface with these monitors, a color is often defined as an \(rgb\) \(triples\) of numbers representing the excitement levels \([17,20]\).

Computer graphics have been defined as “the creation and manipulation of pictures with the aid of a computer” \([17]\). This definition connects the artistic topics of color and color theory to computer science.

This paper is concerned with computer graphics applied to one aspect of color theory, that of color mixture. Three different models of color mixture are commonly discussed: additive, subtractive, and pigmentary.

ADDITIVE MIXTURE
When colored lights are combined, the color of the resultant light is determined by the rules of additive mixture. The color of that light is the sum of the colors of the input lights \([7,13,14,15,21]\).

Since the rgb values represent the amount of light to be physically displayed in each of the primaries, the additive system is easily and naturally applied:

If the colors \((r_1,g_1,b_1)\) and \((r_2,g_2,b_2)\) are mixed in amounts \(m_1\), \(m_2\), the output color \((r_3,g_3,b_3)\) will be equal to their vector addition:

\[
\begin{align*}
r_3 &= m_1r_1 + m_2r_2 \\
g_3 &= m_1g_1 + m_2g_2 \\
b_3 &= m_1b_1 + m_2b_2
\end{align*}
\]

If any one of the terms of the output possesses a value greater than one, the output color has luminance greater than can be displayed on the television monitor. In this
case, corrective action must be taken (see Cook [4] for details).

This algorithm is easily extended to an arbitrary number of colors. The output will be equal to the vector addition of all input colors, within the same provision for correction.

**Subtractive Mixture**

When white light is shone through a series of colored filters, the color of the resultant light is derived according to the laws of *subtractive mixture*. Each filter “subtracts” from the white light the portion of the spectrum which it does not reflect. The final light will consist of only those portions of the spectrum which all the filters reflect [1, 5, 7, 13, 15, 16]. This process corresponds to mathematical multiplication:

\[
\begin{align*}
  r_3 &= (r_1^{m_1}) \cdot (r_2^{m_2}) \\
  g_3 &= (g_1^{m_1}) \cdot (g_2^{m_2}) \\
  b_3 &= (b_1^{m_1}) \cdot (b_2^{m_2})
\end{align*}
\]

This algorithm can be extended to the mixture of an arbitrary number of colored filters by simply extending the number of terms involved in the multiplication. The exponentiation on the components reflects the fact that as the thickness of the filter increases, reflectance decreases proportionally.

**Pigmentary Mixture**

When pigments or dyes are mixed, the color of the resultant surface is determined by pigmentary mixture. This is the model intuitively used by a computer user. The intuitive mixture of blue with yellow is neither white (additive mixture) nor black (subtractive mixture), but rather green, the pigmentary mixture.

A model of pigmentary mixture is proposed which deals in terms of a color’s hue, its saturation, and its lightness (hsl). The hue of a color reflects its basic nature, the saturation its paleness, and lightness the amount of white present in it. Well-defined algorithms for translation between rgb and hsl exist [12, 20].

The hue of a number can be defined according to a variety of differing color wheels [2, 6, 8, 10, 14, 18, 19, 21]. For example, the rgb to hsl translations commonly provided [12, 20], use a circular color wheel with red, green, and blue primaries. Pigmentary mixture, based on a red, yellow, blue primary system, requires a wheel in which these three colors form primaries (i.e. are equally spaced around the wheel at 120 degree angles) [8, 18, 21]. Translations between these two wheels can be made by simple mathematical mappings.

Each color can be uniquely mapped to a point in a hexagonal cylinder. As hue is essentially a modular quantity, the hue of a color corresponds to an angle of location. Saturation represents the proportional length of a line drawn at that angle from the center of a hexagon plane. For example, colors with saturations of 0 and 1 would be located at the center and edge, respectively, of the hexagon. The lightness of the color determines its height in the third dimension of the cylinder. The unique point so constructed will be called that color’s color point.

Varying the hue changes the angle of location of the color point within the hexagon. Varying the saturation changes the distance of the color point from the center. Varying lightness changes the height of the color point in the cylinder.

After two color points have been constructed, one for each color in the mixture, a line can be drawn between them. This line describes all the mixtures of these two colors as their concentrations vary. The color point of the mixture is the point on the line located such that the ratio of the lengths of the two line segments formed is equivalent to the ratio of the amounts of the colors being mixed. For example, if there are equal amounts of color being mixed, the color point of the result will lie at the midpoint of
the line. Once the color point of the mixture has been determined, its rgb value can be determined by inverting the operations described above.

Extension to more than two pigments can be accomplished by regarding the location of the mixture as the center of gravity of the color points of the colors being mixed, where each colorant is given a weight corresponding to its proportional presence in the mixture.

The above model contains many of the same rules as those of Sargent [18] and Von Bezold [1], with two major changes: lightness, not value, is used for the third dimension, and the hexagon, not the circle, is used for the planar figure.

A fundamental property of pigment mixtures is that complementary pigments, in equal proportion, mix to grey [3,8,9,15,20]. In the hue, saturation, and value (hsv) system used by Sargent [17] and Von Bezold [1], all colors of maximum intensity (value) lie on the same plane. For example, red, blue, green, orange, and white all are of full value. If the complementary colors of red and green or blue and orange are mixed, the color point that lies at the center of the plane is that of white, as it possesses no saturation and equivalent value. Thus, mixture in the hsv system fails to account for mixture of complementary pigments.

In the hsl system, all colors of equivalent brightness (l) lie on the same plane. Specifically, grey lies in the center of the primary (red, blue, green, orange) plane, rather than white, as grey possesses the same brightness as the primaries. Mixture of complementsaries is thus perfectly simulated, grey lying at the center of the primary plane.

**Justification of Hexagonal Geometry**

Some previous studies of color [1,5,6,12,18] have used the circle as the planar geometric figure within which to locate colors. However, the circle is not a suitable figure for pigmenary mixture. The mixture of two adjacent fully saturated colors should produce a mixture of full saturation. The midpoint of a line drawn between the color points of those colors will be displaced towards the center of the circle. Therefore, the mixture predicted by a circular model will be abnormally desaturated, because the midpoint of a chord of a circle will never lie on the edge of the circle. Thus, abnormal desaturation will occur in all but complementary mixtures using a circular model.

That problem can be minimized by using a regular n-gon instead of a circle. The midpoint of a line drawn between two points on the edge of an n-gon will often lie on the edge as well.

Note that this will also predict abnormal desaturation on occasion, as when the color points lie on the center of adjacent faces. However, it does this far less often than the circular model. The precise mechanism of computing resultant saturation is still under investigation. Curved lines connecting the color points fare no better than straight ones, as they predict abnormally high saturations for near-complementary mixtures. Straight line mixtures are easily computable for arbitrary color points, and were maintained for this reason.

What value should n take? N must be a multiple of two, so that every color is located symmetrically with respect to its complement. It must be a multiple of three, due to the presence of the three primaries. It should be as small as possible, because as n approaches infinity, a regular n-gon approaches a circle, a figure whose shortcomings have been discussed. Thus, the geometry of the situation implies that a hexagon will best approximate pigmentary mixture.

This geometric conclusion is supported by the findings of Smith [20] that the rgb system lends itself naturally to hexagonal color space, and of Kuppers [14] that the hexagon represents pure color more logically than the circle.
While the model will not predict the precisely correct solution in every case, it is generally believed that no model can \([1,5,7, 13,19,21]\). In nearly every case, however, it provides an excellent approximation.

Specifically, it satisfies each of the three basic principles of pigmented mixture \([3,9, 10,16,21]\): a color mixed with black will lower its lightness, mixed with white will lower its saturation, and mixed with its complement will produce grey.

**IMPLEMENTATION**

These algorithms have been implemented as part of an interactive color mixing/matching/making database program written in the C programming language. This program is presently running at the University of Wisconsin Image Processing and Graphics Laboratory on a PDP-11/45 computer with an STC-70 graphics terminal.

**FURTHER RESEARCH**

Currently, research is being performed to test the applicability and usability of old (e.g. C.I.E. and Kuppers \([14]\)) and new color solids, and the “integrated mixture” algorithm of Kuppers \([14]\).

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**REFERENCES**