THE TEMPERATURE OF THE BOTTOM DEPOSITS OF LAKE MENDOTA; A CHAPTER IN THE HEAT EXCHANGES OF THE LAKE

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In the following paper the observations on Lake Mendota and their discussion are the work of Mr. Birge and Mr. Juday. The mathematical discussion of the data, which adds so much to the value of the paper, is the work of Mr. H. W. March, Associate Professor of Mathematics in the University of Wisconsin.

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Introduction.

Lake Mendota has an area of 39.2 sq. km. and a maximum depth of more than 24 m. Its water is contained in a shallow saucer of fine grained blue clay, underlain by a stratum of glacial drift and overlain by more calcareous lake deposits. This saucer is water-tight except around the margin, where waves have washed out the fine grained clay and left sand and gravel to make the connection with the adjacent shore.

In general therefore, the temperature of the water of the lake is wholly dependent on heat received at or through its surface and the temperature of the bottom deposits is regulated by that of the overlying water. These bottom deposits —hereafter called mud—gain heat from the water during the spring and summer and give it back during autumn and winter. This movement of the heat through the mud is

effected by conduction. This paper deals with this exchange of heat in the upper strata of the mud, to a depth of five meters below its surface.

One reason for this investigation was the desire to complete the study of the heat budget of the lake. How much heat passes through the water of the lake into the bottom deposit during the warming season and comes back during the cooling period? A second and more specific question is perhaps more interesting. During the ice-period the water of the lake gains steadily in temperature, the gain averaging about 20 cal. per sq. cm. of the lake surface per day during winter. How much of this gain comes from the bottom of the lake and how much from other sources? These were the questions which called out the study.

The instruments employed and the methods of using them have already been described in full[1] and the account need not be repeated. The instrument was an electrical resistance thermometer, mounted in a brass case and attached to the end of a long iron pipe which could be driven into the mud. The first thermometer used could reach a depth of 3.5 m.; later a longer pipe was used, reaching a depth of 5 m. An insulated cable extended up from the resistance coil through the pipe and to the surface, where the resistance was read by the indicator box of a Whipple thermophone. This gave the reading directly in degrees centigrade, and the several thermometers were repeatedly and carefully compared with a standard thermometer.

The total length of the thermometer is more than six meters and it was not often used in water of less depth than 7.5 m.–8 m. This was due mainly to the increased density of the bottom deposits near the shore and within the limits of wave action. The metal point containing the resistance coil was joined to the iron pipe by a wooden connection so as to interpose a non-conducting unit between coil and pipe. Since this necessarily limited the force of the blows which could be used in driving the pipe, the thermometer could not be forced far into sandy bottoms and the attempt was seldom made. At the Eight-meter and Twelve-meter stations the lake deposits are so thin that the thermometer goes

[1] A second report on limnological apparatus. E. A. Birge. Trans. Wisconsin Academy 20; 533–551. Madison, 1922.

through them into the stiffer glacial clay. This was shown by the fact that the penetration was sometimes stopped by encountering a boulder. In the deep water the marly lake deposits are 10 m. or more thick, above lake clays whose thickness aggregates more than twice as much.

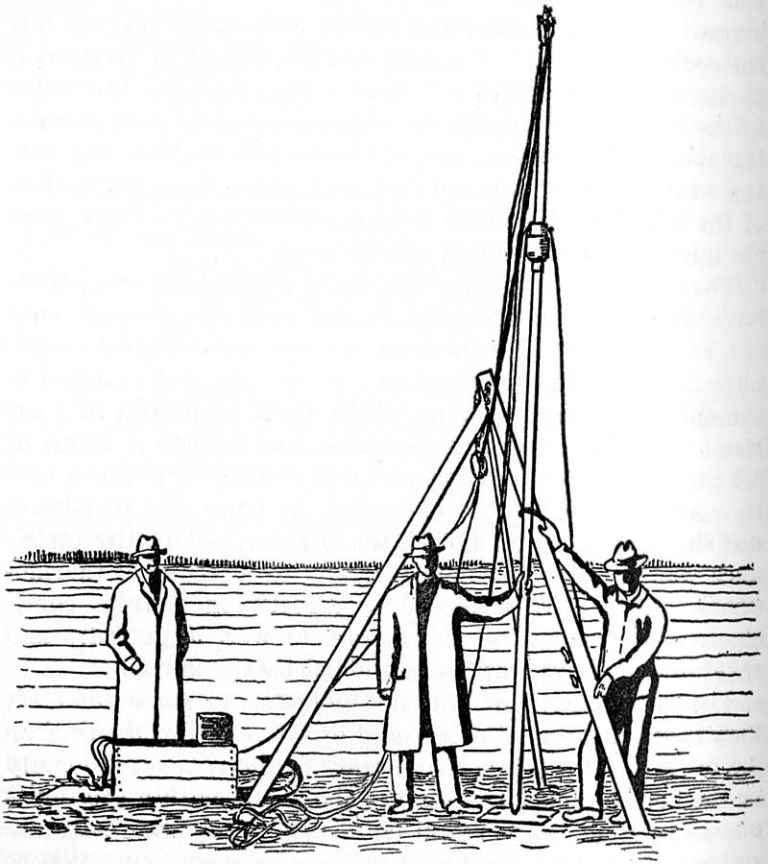

Fig. 1. The mud thermometer on the ice. This is the first one made, the 3.5 m. instrument, with hammer permanently attached to it. The insulated wires and rope are seen attached to the top; also the hammer with its two lines. The thermometer is driven into the mud as far as the point where the hammer rests. From Trans. Wis. Acad.; 20, 534, Madison, 1922.

All these deposits are dense. In the surface strata the organic matter may amount to about 10–15 percent of the dry weight, the remainder consisting largely of calcium carbonate and silica. These materials are combined into a

pasty mass containing about 84 percent of water near the surface. The whole mud thermometer weighs 40 kg., including the hammer of 10 kg. At the station in the deepest water, where the lake deposits are softest, the instrument will penetrate 1.5 m.–2 m. by its own weight. The resistance becomes greater with depth; at first three or four blows of the hammer will drive the thermometer down a half-meter, but when it has reached a depth of perhaps 4 m. 20 or more blows of the hammer are needed to force it to 4.5 m. At the stations nearer shore this resistance not infrequently put a stop to further penetration. This stiff and pasty condition of the mud also made the withdrawal of the thermometer no easy task. The hammer had to be brought into use to jar it loose by striking the top from below and this procedure caused the loss of one thermometer by breaking off the top.

Data

Regular observations were made at four stations in Lake Mendota.[2] The first, known as the Eight-meter station, is near the south shore of the lake, about 250 meters from the University shore in eight meters of water. The second station, the Twelve-meter station, is just beyond it. Both are on the rather steep slope of the lake bottom which here falls off from a depth of 5 m. to 15 m. in a distance of 200 m. to 300 m. The third station, Station I, is at the depth of 18 m. in the deepest water found in University Bay and about one kilometer from the south shore of the lake. This basin is somewhat separated from the main basin of the lake (p. 206). The outer station, Station II, is about two kilometers from shore in 23.5 m. of water. It is in the southern part of the main basin of the lake and the water here is practically at its deepest. Stations I and II are the points where observations of the temperature and plankton of Lake Mendota have been made for more than thirty years.

The number of series of observations differs somewhat at the several stations as will be seen from the following table.

[2] See map in Juday, C., Inland lakes of Wisconsin. Bull. Wis. Geol. Survey, 27, 14. Madison, 1914.

TABLE 1. *Number of series of observations on mud temperatures.*

Station	1916	1917	1918	1919	1920	1921	Total
8 m.	-------	12	13	19	17	2	63
12 m.	1	4	12	21	13	2	53
I	1	5	12	15	11	2	46
II	1	5	13	14	10	2	45
	3	26	50	69	51	8	207

In this table are included the observations made at the regular stations and those made at corresponding depths. The work began with a trial series taken through the ice in the spring of 1916; it was resumed in Jan. 1917, was continued through the ice period, and taken up again in the latter part of the year. In 1918, 1919, and 1920 observations were continued throughout the year and the work was closed by series taken in January and February 1921.

The observations made at the regular stations are recorded in the tables of the appendix.

RESULTS

The results of the observations at the several stations will be discussed mainly on the basis of the mean of the three years 1918, 1919, and 1920. Curves were constructed for each recorded depth in the mud (Bot., 0.5, 1 m., etc.) by platting the observations on coordinate paper at the appropriate date and connecting successive observations by a straight line. The mean of the three sets was taken where the several lines cross the date lines for the first and the fifteenth of each month. From these means are derived the data of the tables and the diagram for each of the four stations.

Smooth average curves can not be expected from so small a number of observations taken through only three years. But since the movement of temperature in the mud is necessarily slow and since the temperature of the water also varies slowly the mean results are not unsatisfactory. In order to give opportunity for comparison with the data, two diagrams are given for single years, one at the Eight-meter station and the other at Station II. Fig. 4 gives the

results at the Eight-meter station for 1919 and 1920, and fig. 10 shows the same facts for Station II (23.5 m.) for 1918 and 1919. Comparison with the diagrams representing the mean results shows that the general run of the lines is closely similar. It is especially instructive to note

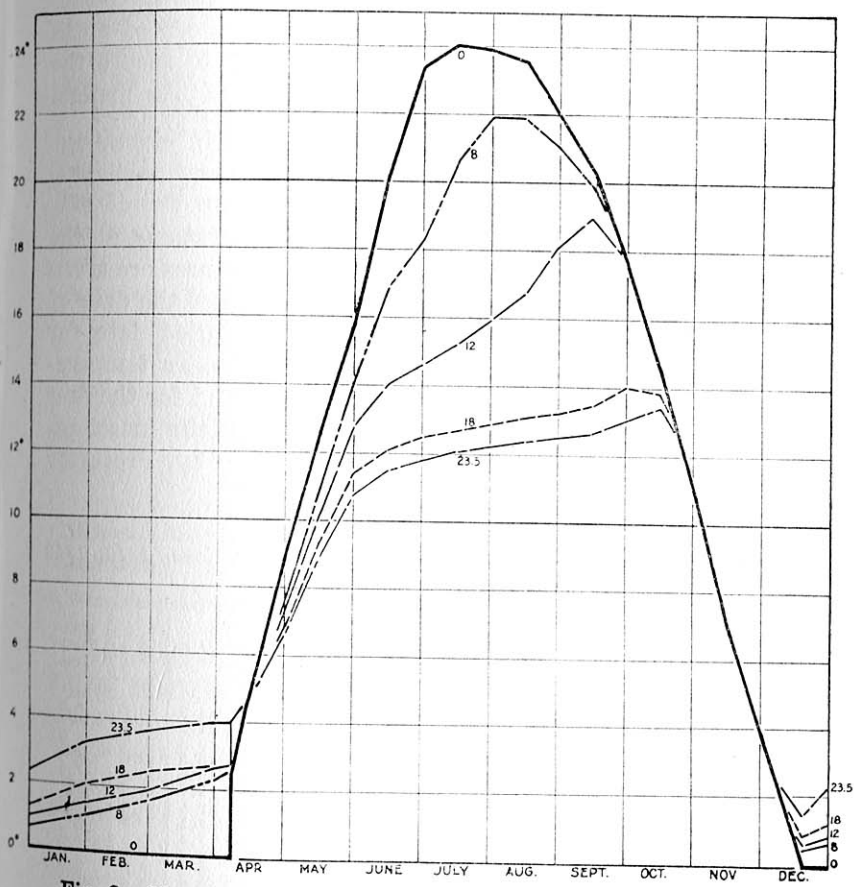

Fig. 2. Mean annual course of temperatures at various depths in the water of Lake Mendota. They represent the results of observations carried on for 16 years, during a period which ended in 1916 and therefore does not include the years when the studies were made on mud.

the dates at which the temperature lines for the several depths cross each other.

The temperatures were read in 1918 to a depth of 3.5 m. and often to a somewhat greater depth; so that the mean

for 4 m. in the several tables was derived from three years, by extending the curves for 1918 a half-meter or less. The mean for 5 m. comes from two years only, but this fact makes little difference in the general result.

The mean temperatures for the first and the fifteenth of each month as given in the tables were platted on coordinate paper and connected by lines; the intercepted areas were measured with a planimeter in order to secure the mean temperature of the mud to a depth of five meters. Some of these curves are shown in figs. 5 and 11.

The Eight-meter Station

Table 2 gives the mean results of observations at the Eight-meter station and the detailed temperatures are given in the appendix (p. 231). Fig. 3 gives part of the data of table 2 in a diagram; fig. 4 shows the detailed data for 1919 and 1920; fig. 5 shows the curves of mean temperature of the mud to the depth of five meters and for the first of each month. Table 16 and fig. 13 show the mean annual course of temperatures as computed by Professor March.

Table 2. *Eight-meter Station. Mean of observations, 1918, 1919 and 1920.*

Depth	Bot.	0.5	1.0	1.5	2.0	2.5	3.0	4.0	5.0	Mean Temp.
Jan. 1	0.8	4.2	6.5	8.4	9.8	10.7	11.0	11.3	10.8	8.9
Jan. 15	0.9	3.6	5.9	7.7	8.8	10.2	11.6	11.0	10.6	
Feb. 1	1.1	3.6	5.6	7.3	8.4	9.8	10.2	10.8	10.6	8.2
Feb. 15	1.3	3.4	5.1	6.8	7.9	9.1	9.7	10.5	10.6	
March 1	1.5	3.1	5.0	6.4	7.5	8.3	9.1	10.2	10.6	7.5
March 15	2.0	3.3	4.7	6.0	7.2	7.9	8.7	9.7	10.5	
April 1	2.7	4.1	4.9	6.0	7.1	7.8	8.6	9.5	10.3	7.3
April 15	4.1	5.1	5.6	6.2	7.2	7.8	8.5	9.5	10.2	
May 1	6.3	6.2	6.1	6.6	7.2	7.8	8.5	9.3	10.1	7.8
May 15	9.4	8.5	7.4	7.1	7.3	7.9	8.4	9.3	9.9	
June 1	13.5	10.2	8.9	8.1	7.9	8.0	8.4	9.2	9.8	9.0
June 15	16.0	12.8	10.5	9.3	8.5	8.3	8.5	8.9	9.5	
July 1	18.2	14.6	12.2	10.3	9.4	9.0	8.6	8.9	9.4	10.3
July 15	19.9	17.1	14.3	11.7	9.8	9.6	8.9	8.8	9.2	
August 1	22.3	20.1	16.5	13.3	11.6	10.3	9.7	9.2	9.4	12.3
August 15	22.0	19.7	16.8	14.1	12.7	11.1	10.2	9.5	9.6	
Sept. 1	20.0	18.3	16.5	14.4	12.9	11.5	10.7	9.9	9.7	12.8
Sept. 15	18.6	17.3	16.1	14.5	13.1	11.9	11.1	10.0	9.5	
Oct. 1	16.7	16.5	15.5	14.4	13.3	12.2	11.4	10.3	9.5	12.7
Oct. 15	14.4	15.2	14.8	14.2	13.5	12.8	11.8	10.8	9.9	
Nov. 1	12.0	12.9	13.7	14.0	13.5	13.0	12.1	10.9	10.4	12.3
Nov. 15	7.9	9.0	12.0	12.5	13.0	12.7	12.2	11.2	10.6	
Dec. 1	4.6	7.0	10.0	11.2	12.0	12.1	11.9	11.1	10.7	10.4
Dec. 15	2.3	5.7	8.3	9.8	11.0	11.4	11.5	11.1	10.6	

Note. In this and in all similar tables **Bot.** indicates the junction of mud and overlying water.

The minimum mean temperature (7.2°) came on March 15; the maximum (12.8°) came on September 1. The result for Aug. 15, Sept. 15, and Oct. 1 differs from that of Sept. 1 by less than 0.1° and all four may be regarded as practically identical. During this period the temperature of the bottom water falls from 22.0° to 16.7° but the increase in the lower strata of the mud compensates for the loss at the surface.

The observations in each year are so few that it is by no means probable that they represent accurately the minimum and maximum temperatures of the mud. If, however, the lowest recorded series of each year is compared with the highest the following table will result:

TABLE 3. *Eight-meter Station. Minimum and maximum temperatures of mud to the depth of five meters.*

Source	Minimum		Maximum		Difference	Budget, cal.
	Date	Temp.	Date	Temp.		
Table 6	March 15	7.2	Sept. 1	12.8	5.6	2800
Single Years						
1917	March 31	7.1	Sept. 15	13.1	6.0	3000
1918	March 9	6.6	Sept. 13	13.0	6.4	3200
1919	April 18	7.7	Sept. 4	12.9	5.2	2600
1920	March 20	6.9	Aug. 20	12.9	6.0	3000
Mean of single years		7.1		13.0	5.9	2950

The annual heat budget derived from the mean of the single years is larger than that derived from the table of mean temperatures. This result is to be expected since the several years are never exactly alike in the movement of temperature and maxima and minima do not fall on the same date. They thus tend to cancel or reduce each other in the mean. This situation is even more strongly emphasized in the records from greater depths of water, as will appear on later pages.

The form of the curves shown in fig. 3 is worth notice. The movement of the temperature at 0, i. e. the surface of the mud, is along a curve which is substantially symmetrical so far as the part above 4° is concerned. This temperature is reached about April 15; after 3.5 months the maximum temperature of 22.3° is reached and is held for about two weeks. Then follows a rapid decline, reach-

ing 4° again about December 1 after 3.5 months. That part of the curve between December 1 and April 15 has a different form, due to the fact that the lake is covered with ice during most of this period. The temperature of the water changes slowly beneath the ice, and wind-caused circulation is quite cut off.

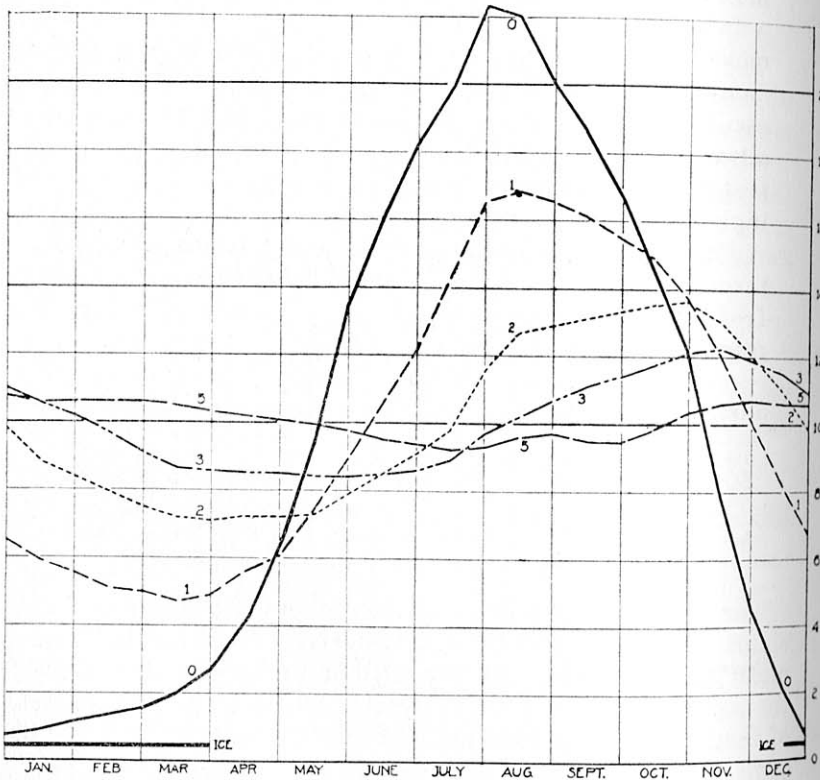

Fig. 3. Eight-meter Station. Annual temperature curves from table 2. Numbers on the curves show depth in meters below the surface of the mud. The line for 4 m. would lie between those for 3 m. and 5 m. It is omitted because it would crowd the diagram. Compare figs. 6, 8, 9; also fig. 13.

This curve, then, is a graphic representation of the annual heat wave in the water of the lake at the depth of 8 m.—a wave which has a period of one year and an amplitude of 21.5° (0.8°–22.3°). Its crest comes in August and its hollow in December. In any single year its form

would be less symmetrical than that shown in the diagram (see fig. 4), since alternation of warm and cold periods would accelerate and delay its advance and retreat. This heat wave passes from the water downward into the mud and returns again, transmitted by conduction; and the successive curves marked 1, 2, etc. show its form at the meter intervals below the surface. The general form of the wave with that of its main irregularities is preserved as it thus moves through the mud; but all small variations are damped out as it proceeds. The amplitude of the annual oscillation, or the swing of the wave, decreases as the wave advances through the mud; and since conduction in the mud is slow the advance and retreat of the wave are correspondingly retarded.

The decrease in amplitude of the heat wave and its retardation as it passes through the mud may be derived from table 2 and are seen in fig. 3. The results are summarized in table 4.

TABLE 4. *Eight-meter Station. Annual oscillation of temperature.*

Depth, meters	Annual oscillation	Amplitude	Per cent	Middle point	Date up	Interval, days	Date down	Interval, days
Bot.	0.8–22.30	21.5°		11.6°	May 21		Oct. 28	
1	4.7–16.8	12.1	57	10.8	June 18	28	Nov. 24	27
2	7.1–13.6	6.5	54	10.3	July 21	33	Dec. 23	29
3	8.4–12.2	3.8	59	10.3	Aug. 17	27	Jan. 23	31
4	8.8–11.2	2.4	63	10.0	Sept. 15	29	Mar. 8	44
5	9.2–10.7	1.5	62	10.0	Oct. 22	37	May 8	61

Notes. **Amplitude** is the total swing of the heat wave in degrees. **Percent** is the percent of the amplitude at the given meter, taking that of the meter above as 100. **Middle point** is the temperature at the middle of the up-swing or down-swing. **Interval** is the period in days by which the heat wave is retarded in passing from meter to meter in the mud.

The results in this table show some irregularity, as would be expected; those from the upper meters are more trustworthy than those from the lower strata, since the movement of temperature is greater and more rapid and the variations of individual observations make less difference. The mean percent of amplitude in the upper three meters is about 57; in other words the amplitude of the heat wave is reduced by about 43 percent in passing through one meter. The percentage of transmission seems higher at 4 m. and 5 m. This may be due to the greater

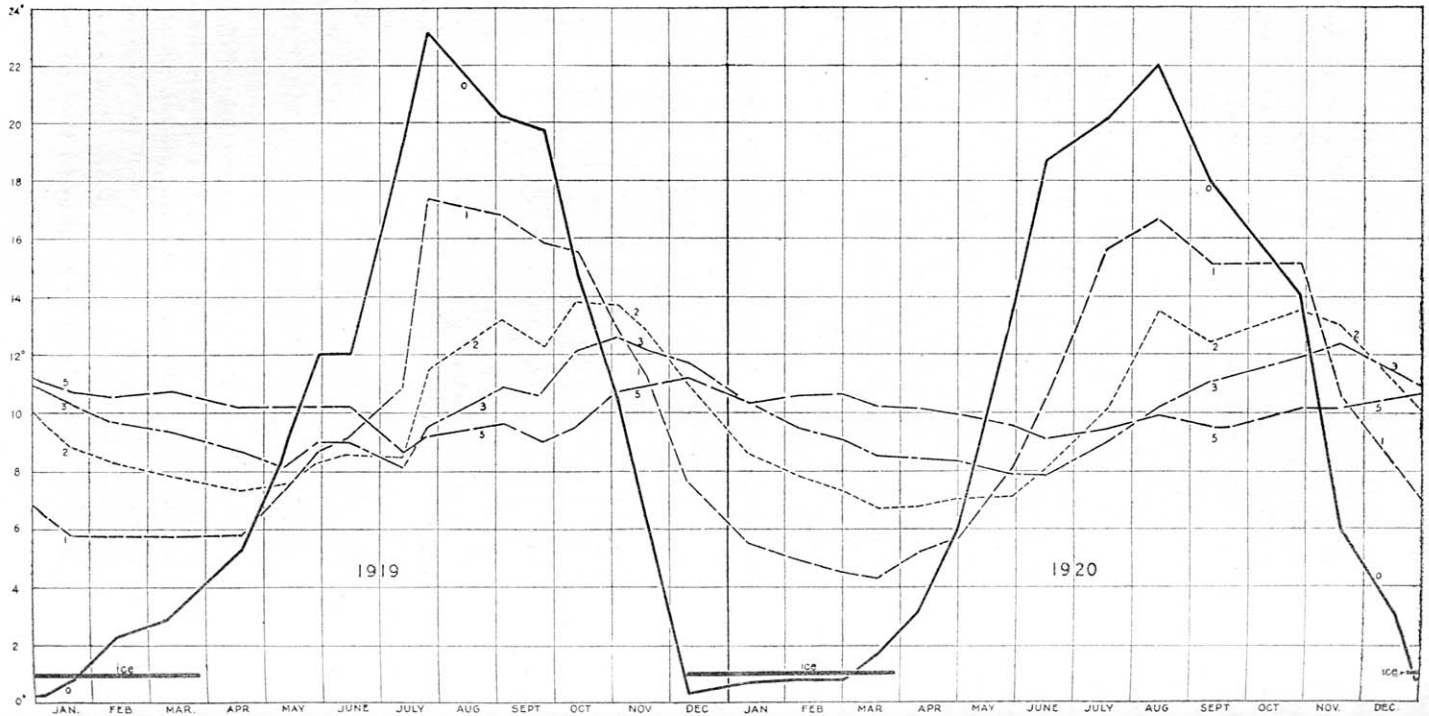

Fig. 4. Eight-meter Station. Result of platting the series of observations for 1919 and 1920. Note especially the different run of the curves in the two winters and the marked variation in July, 1919. See p. 219.

density of the mud as depth increases but it may also be an accidental result. Probably a mean transmission of 57 percent should be assumed through the whole depth.

The mean amount of phase shift or lag in the downward progress of the heat wave in passing from meter to meter is about 31 days. The variations from the mean are no larger than would be expected from so few observations. The same is approximately true of the upward movement of the wave—the progress of the cooling of the mud—so far as the upper three meters are concerned. In the case of meters 4 and 5, there is much delay in the fall of temperature—the upward movement of the heat wave. This is probably due to the effects of the ice period, which slows up all temperature changes; but in part the difference may also be occasioned by accidental variations. (See p. 214). We may conclude that there is a lag of about 30 or 31 days in the movement of the heat wave from one meter to the next through the mud.

These results, with the data of table 2 and the curves of fig. 3, should be compared with those of table 16 and fig. 13, which give the computed results from a smoothed temperature curve at the surface of the mud. Such treatment eliminates accidental or occasional variations. There is a close fit between the observed and computed forms of the heat wave at the surface. The observed curves for 1 m. and 2 m. show sharp rises in mid summer which carry them above the computed curves. The fit for the 3 m. curve is the best; that observed for 5 m. rises more rapidly in late October and in November than does the computed curve. On the whole there is a satisfactory agreement between the two sets of data.

The slow conduction of heat through the mud with the accompanying lag in its propagation greatly complicates the process of the total heat exchanges of the five-meter stratum which was investigated.

Table 2 for instance, shows that in the six months between Jan. 1 and July 1 the five-meter stratum gained 1.4°; more accurately stated, the gain was 730 calories per square centimeter of surface. Fig. 5 shows that the stratum gained heat to the depth of nearly 2 m. and lost heat from that depth to 5 m. and the loss extended to a greater depth.

Above 2 m. the mud gained heat amounting to about 1310 cal. per square centimeter; between 2 m. and 5 m. it lost about 580 cal.; leaving a net gain of 730 cal. The mean temperature on June 1 was almost exactly the same as on Jan. 1. There was a gain of about 500 cal. above 1.5 m. (fig. 5) and a slightly smaller loss below. Mean tempera-

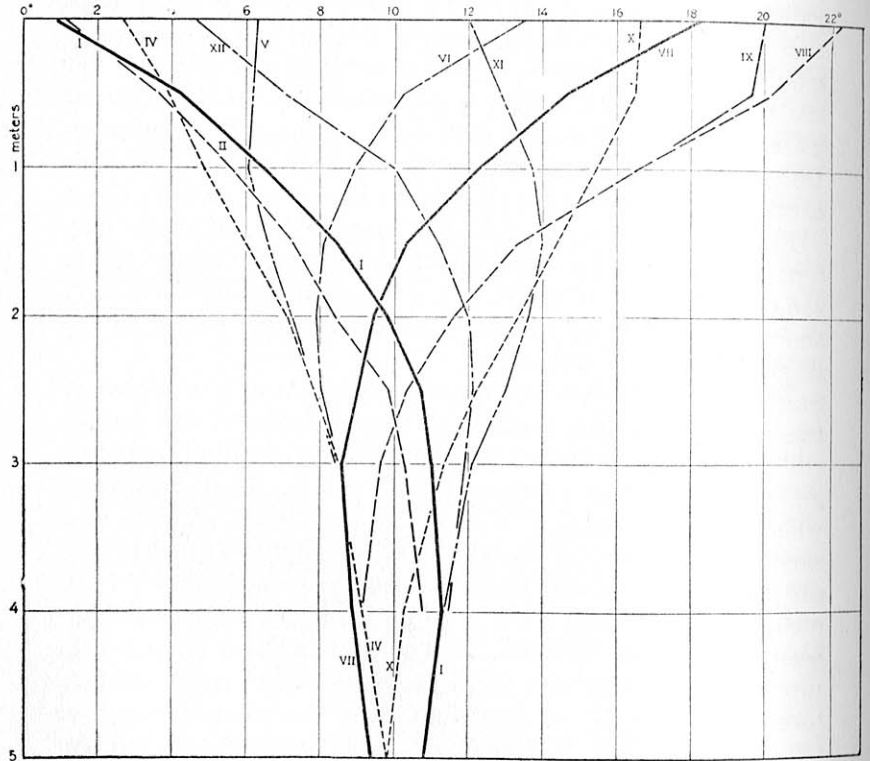

Fig. 5. Eight-meter Station. Mean temperature curves of mud to the depth of 5 m. Data from table 2, taken for the first of each month. Temperatures at dates six months apart are indicated by lines of similar character. The line for March is omitted and the lower parts of other lines are not carried out, in order to avoid excessive crowding of the diagram. The lines bring out with especial clearness the effect of the lag in the penetration of the heat. See p. 199 and compare fig. 11.

tures on July 1 and Dec. 1 are almost equal. Figure 5 shows that during the five months there was a loss of 840 cal. above 1.5 m. but this was slightly overbalanced by gains below that depth. Similar comparisons may be made

for other months and in almost every case the mean temperature will be the algebraic sum of gains in some part of the stratum and losses in other parts.

July seems to be the only month in which gains are made at all depths down to 5 m.; and in November and December heat is lost from all depths. Hence there is a more rapid gain of heat in July than at any other time; more than one-third of the total gain of heat being made in that month. Nearly the same amount is lost in November; but the rate of loss decreases in December, partly because the water reaches the temperature of maximum density about December 1 and partly because the freezing of the lake checks all heat exchanges.

Figs. 3 and 4 show that the curve of temperature at the 5 m. level of the Eight-meter station oscillates near a mean somewhat above 10.0°. Comparison of figs. 6, 8, and 9 shows that the similar mean at the Twelve-meter station is about 9.3°; at Station I (18 m.) it is about 8.3°; at Station II (23.5 m.) about 8.1°. The ground water in this region has a temperature of about 8.3° to 9.5°. Merrill's Springs are large, freely flowing springs on the margin of Lake Mendota. Their temperature is about 8.5°–9.0° when taken in the water rapidly flowing up through the sand. The temperature of the ground at the depth of 5 m. can hardly be higher and it is plain that at the Eight-meter station the mud to a depth of more than 5 m. has been permanently warmed by the lake water to a temperature one or two degrees higher than the normal temperature of the soil.

The Twelve-meter Station

The Twelve-meter station lies only a short distance to the north of the Eight-meter station. Both are on the relatively steep slope of the lake, the depth increasing in that region from 5 m. to 15 m. in a distance of 200 to 300 m. Although so near in both distance and depth there is a great difference between the two stations in the course of the warming of the bottom water. Fig. 2 shows the average curves of temperature in the open lake at depths corresponding to those of the several stations, and the dif-

ference between the curve for 8 m. and that for 12 m. is obvious. The top of the thermocline lies above 12 m. until September, and after June 1 the rise of temperature at 12 m. is much retarded. On the other hand, the 12 m. level is only a small distance below the thermocline and temperatures at that depth and near the shore are much influenced by wind. An illustration of this influence is found in the high temperature recorded at the surface of

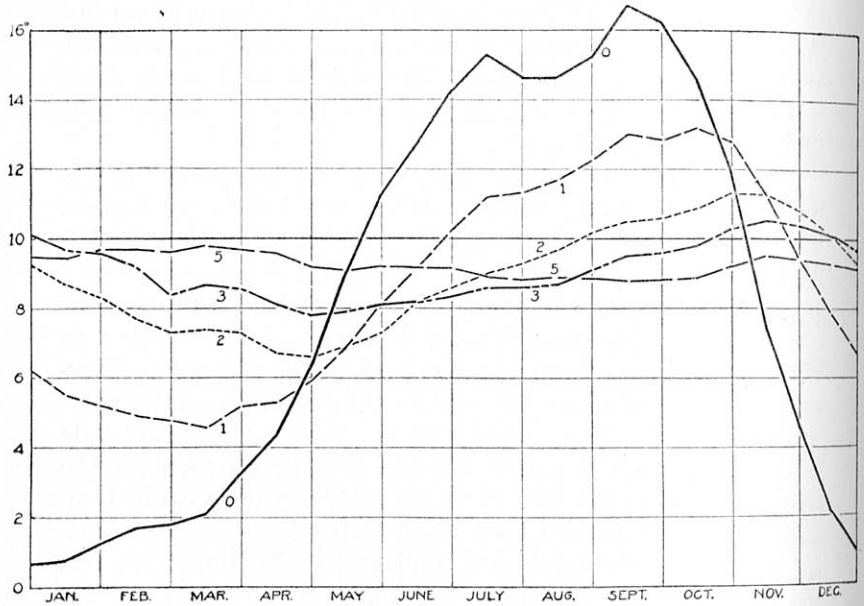

Fig. 6. Twelve-meter Station. Annual temperature curves from table 5. The marked elevation shown in July on the line representing the surface of the mud (marked 0) is due to wind effect at time of observations. The normal curve for the depth of 12 m. is given in fig. 2. Note that this curve in fig. 2 shows a higher temperature than does that of fig. 6. The water temperatures at all depths below the thermocline during the years of observations on the mud were below the average.

the mud on July 12, 1919 (tab. 18) ; and a fall brought about by the same means is seen on Aug. 16, 1920. To the first case is due much of the elevation seen on the upper curve in fig. 6 at July 1, followed by a depression. This irregularity does not appear in the temperatures for the mud at depths of 1 m. or more, since the high temperature does not last long enough to have a perceptible effect.

Thus the temperature curve for the surface of the mud is much flattened during the summer by the slowness with which the heat reaches the 12 m. level. This situation is reflected in the curves for deeper temperatures, which are much flatter than corresponding curves at the Eight-meter station.

In September the thermocline reaches the depth of 12 m. and there is a corresponding rapid rise of temperature in the water. But this rise comes so late in the season that it is quickly overtaken by the autumnal decline in temperature and so has little effect on the temperatures of the mud.

Table 5 shows the mean temperatures for the Twelve-meter station in the same way that table 2 shows them for the Eight-meter station. Fig. 6 shows the run of the main curves as derived from table 5 and fig. 7 shows the relations of the temperatures of the water to those of the mud on certain dates of 1920.

TABLE 5. *Twelve-meter Station. Mean of observations, 1918, 1919, and 1920.*

Depth	Bot.	0.5	1.0	1.5	2.0	3.0	4.0	5.0	Mean Temp.
Jan 1	0.7	3.0	6.2	8.3	9.3	10.1	10.1	9.6	8.2
Jan. 15	0.8	3.1	5.5	7.6	8.7	9.7	9.8	9.6	
Feb. 1	1.3	3.2	5.2	7.2	8.3	9.6	9.8	9.7	7.7
Feb. 15	1.7	3.3	4.9	6.7	7.7	9.2	9.6	9.6	
Mar. 1	1.8	3.4	4.8	6.2	7.3	8.4	9.2	9.6	7.1
Mar. 15	2.1	3.8	4.6	6.2	7.4	8.7	9.3	9.8	
Apr. 1	3.3	4.3	5.2	6.2	7.3	8.6	9.3	9.7	7.4
Apr. 15	4.4	4.9	5.3	5.9	6.7	8.1	9.0	9.6	
May 1	6.3	6.3	5.9	6.1	6.6	7.8	8.8	9.2	7.4
May 15	9.1	8.0	6.9	6.7	6.9	7.9	8.8	9.1	
June 1	11.3	9.7	8.2	7.5	7.3	8.1	8.6	9.2	8.4
June 15	12.7	10.8	9.2	8.5	8.2	8.2	8.6	9.2	
July 1	14.2	11.9	10.3	9.3	8.6	8.4	8.6	9.1	9.4
July 15	15.3	12.8	11.2	9.9	9.0	8.6	8.7	8.8	
Aug. 1	14.6	12.9	11.3	10.1	9.3	8.6	8.7	8.8	9.9
Aug. 15	14.6	13.3	11.7	10.3	9.7	8.7	8.8	8.8	
Sept. 1	15.2	14.0	12.3	11.2	10.2	9.1	9.0	8.9	10.4
Sept. 15	16.7	16.0	13.0	11.5	10.5	9.5	9.2	8.8	
Oct. 1	16.2	15.5	12.8	11.6	10.6	9.6	9.2	8.8	10.8
Oct. 15	14.2	14.9	13.2	12.0	10.9	9.8	9.4	8.9	
Nov. 1	11.9	12.2	12.8	12.4	11.3	10.3	9.5	9.2	10.8
Nov. 15	7.4	10.5	11.3	11.7	11.3	10.5	10.1	9.5	
Dec. 1	4.6	7.8	9.4	10.5	10.8	10.4	10.0	9.4	9.6
Dec. 15	2.2	5.5	7.9	9.4	10.1	10.1	9.5	9.3	

The heat budget of the mud at the Twelve-meter station is smaller than at the Eight-meter, corresponding to the smaller annual oscillation of temperature in the bottom water. Table 6 gives the facts in form similar to that of table 3. No budget is possible for 1917 as no reading was made at 12 m. in September of that year.

TABLE 6. *Twelve-meter Station. Minimum and maximum temperatures of the mud to the depth of five meters.*

Source	Minimum		Maximum		Difference	Budget in calories
	Date	Temperature	Date	Temperature		
Table 9	Mar. 15	7.1	Sept. 1	10.9	3.8	1900
Single years						
1918	Mar. 23	6.3	Sept. 15	12.1	5.8	2900
1919	April 18	7.5	Sept. 23	10.6	3.1	1550
1920	Mar. 20	6.7	Oct. 29	11.0	4.3	2150
Mean		6.8		11.2	4.4	2200

In general the same comment may be made on this table as on table 3. The heat budget at 12 m. is about two-thirds as great as that at 8 m. The budget derived from the mean of the single years is larger than that from the table of mean temperatures. A notable difference between the two tables is the far greater range of variation in the budgets of the single years at 12 m. as compared with those at 8 m. The smallest budget in table 3 is 80 percent of the largest; in table 6 it is little more than half as large. This is due to the varying rate of penetration of heat into the lake in different years, especially in the earlier part of the open season. In 1917 and 1918 the water below the thermocline was warmer, in 1919 and 1920 it was colder. This relation holds for all depths below the thermocline, so that the variation in annual budgets is even more marked at Station I and Station II than at the Twelve-meter station. There is much less variation in the temperature of the epilimnion than in that of the lower strata and this fact accounts for the uniformity of the budgets at the Eight-meter station.

The mean annual oscillation of temperature in the bottom water (table 5) is 16° (0.7°–16.7°), about three-fourths of that at 8 m. But nearly 1.5° of this rise comes in September, so late that it is unable to exert its full effect on the temperature of the mud. Hence the total budget is more nearly comparable with that from an annual oscillation which should omit the autumnal peak.

The transmission of the annual oscillation to depths of 1 m. and 2 m. averages about 50 percent per meter. It is therefore lower than at the Eight-meter station. The

deeper meters show a higher transmission, though it varies greatly from meter to meter. This is also true of stations I and II. The probable cause of the smaller transmission in the upper strata is the smaller conductivity of the softer mud.

Figure 7 is given to show the relations between the temperatures of water and mud at various dates in 1920. The curve for January shows an early winter condition be-

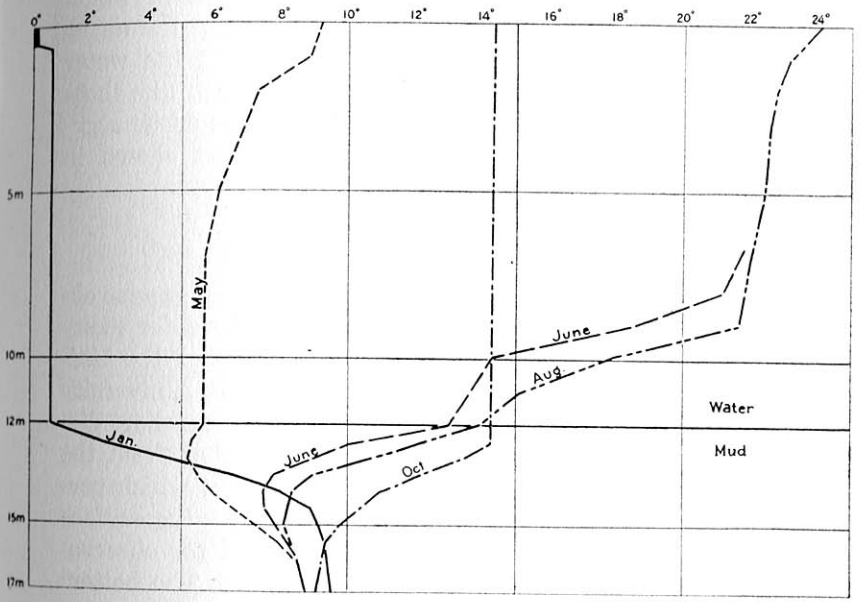

Fig. 7. Typical curves showing temperatures of water and mud from single observations of 1920. Note the great lag of penetration of water temperature into the mud. (see p. 199); and the consequent movement of temperatures in opposite directions in different strata of the mud. The line for May shows the beginning of the thermocline, which is fully developed in June and August in the stratum between 9 m. and 12 m. In October the lake has become homothermous at least to the depth of 12 m.; the upper stratum of the mud is cooling while the deeper layers are still gaining heat.

neath the ice. The line for May shows a mid-spring situation in the water, when the rapid advance in the warmth of the whole mass of water of the lake has been checked and the thermocline is beginning to be indicated in the surface strata. The mud shows warming at the surface and cooling in the deeper strata. The lines for June and

August show a fully developed thermocline below the depth of 9 m. in the water. The surface strata of the mud are at their maximum but the lower part is still much below the temperature which will be reached later. In October the epilimnion has cooled and grown thicker until it has reached the bottom at 12 m. The surface of the mud is cooling with the bottom water, while the lower strata are still warming.

A similar diagram for the Eight-meter station would give temperature curves in the water much like the upper 8 m. of those in fig. 7. At stations I and II the water temperatures in the upper 12 m. would be much like those of fig. 7; while in the lower water the lines would be regularly brought down to meet the temperatures shown in tables 7 and 9 at the proper depth and date.

Station I—18 Meters

Station I was occupied for mud temperatures because observations of other types have been made there for many years. It lies about one kilometer northwest of the University shore, near the bottom of the basin of University Bay. This reaches a maximum depth of 20 m. a short distance to the west of Station I; it is separated from the main basin of the lake by a low and flat ridge which perhaps rises to a maximum height of 17 m. below the surface of the water. The station proved less suited for observation on mud than any of the other stations. The bottom at this point has a very slight slope so that which we have called "drainage" in winter is not active (p. 215), and on the other hand it is too far from the 20 m. spot to get the full benefit of the warmed water which in winter settles down into the lowest parts of all hollows in the bottom. The result was an amount of irregular variation in the readings, especially in winter, which were proportionally greater than at other stations, with corresponding unevenness in the means.

Table 7 shows the facts of the mean temperatures as derived from the years 1918, 1919, and 1920. The range of oscillation is much smaller than at stations in the shallower water, and therefore only the temperatures for the first of each month are printed in the table.

TABLE 7. *Station I—18 meters. Mean of observations, 1918, 1919, and 1920.*

Depth	Bot.	0.5	1.0	1.5	2.0	3.0	4.0	5.0	Mean temp.
Jan. 1	1.8	4.4	6.4	7.7	8.5	8.8	8.5	8.6	7.5
Feb. 1	2.5	4.5	6.1	7.0	8.0	8.9	8.6	8.6	7.4
Mar. 1	3.4	5.0	6.3	7.3	7.8	8.8	8.7	8.7	7.5
Apr. 1	4.1	5.3	6.2	6.9	7.5	8.4	8.6	8.8	7.4
May 1	6.3	6.3	6.2	6.6	7.2	8.1	8.4	8.6	7.3
June 1	9.6	8.2	7.2	7.2	7.5	8.0	8.4	8.6	7.9
July 1	10.8	10.0	8.8	7.9	7.7	8.0	8.2	8.4	8.3
Aug. 1	11.4	10.6	9.8	8.9	8.4	8.0	7.9	8.1	8.6
Sept. 1	11.8	10.7	9.9	9.0	8.5	8.1	7.8	7.9	8.7
Oct. 1	12.5	11.0	9.8	9.1	8.6	8.1	7.8	7.8	8.7
Nov. 1	10.7	10.5	9.8	9.3	8.9	8.2	7.8	8.1	8.7
Dec. 1	4.6	7.5	8.6	8.7	9.0	8.7	8.0	8.1	8.1

Fig. 8 shows the movement of temperature at Station I, as derived from table 7. The annual oscillation in the bottom water shows in an exaggerated form the same contour as that at 12 m., fig. 6. It has a rapid rise in spring, a very slow rise during summer and early fall, amounting

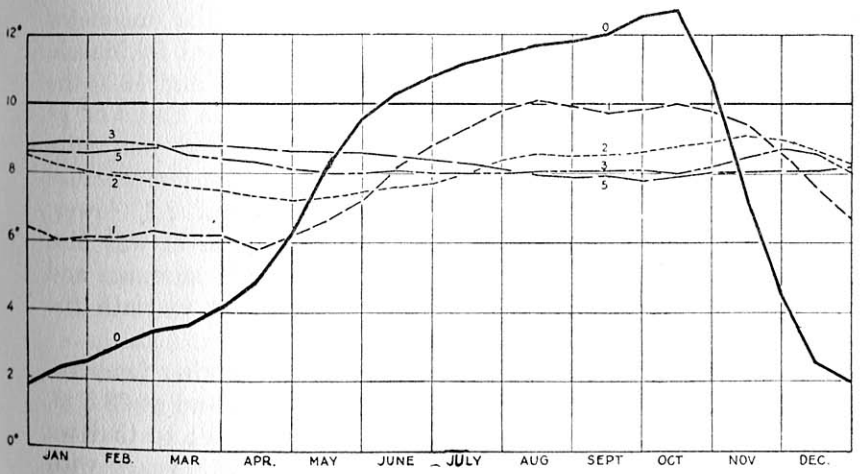

Fig. 8. Station I, 18 m. Annual temperature curves from table 7. Note characteristic temperature curve of deeper water in Lake Mendota, indicated by line marked 0. The flat curve during summer is repeated in the lines for the several meters. Compare figs. 3, 6, 9.

to only 2.5° in 3.5 months. There is a rapid but small rise in late September and early October, as the epilimnion cools and grows thicker until it reaches the depth of 18 m. This rise is promptly cut off by the great autumnal down swing of temperature, and has little effect on the mud.

The annual heat budgets are shown in table 8, which should be compared with tables 3, 6, and 10.

TABLE 8. *Station I—18 meters. Minimum and maximum temperatures of mud to the depth of five meters.*

Source	Minimum		Maximum		Difference	Budget in calories
	Date	Temperature	Date	Temperature		
Table 7	April 15	7.3	Oct. 1	8.7	1.4	700
Single years						
1917	Mar. 21	6.8	Sept. 15	9.1	2.3	1150
1918	Mar. 23	7.2	Oct. 22	9.7	2.5	1250
1919	April 19	7.4	Oct. 11	8.4	1.0	500
1920	Mar. 13	6.7	Oct. 25	8.6	1.9	900
Mean		7.0		8.9	1.9	950

The irregularities in these heat budgets are plain; the smallest is only 40 percent of the largest. The cause may be seen from the details in table 19. In 1917 and 1918 there was a steady fall of temperature at the one-meter level during the winters, reaching a low level in March. The maximum fall was about 1.5°. In 1919 and 1920 the temperature at this level remained within the limits of 1° during the first three months of the year and was quite as high in March as in January. In 1919 the temperature lay close to 7.5° while in 1920 it was about 2.2° lower. This higher temperature at the one-meter level was also present at lower levels and since the late summer and autumn months did not bring any unusual warmth the budget of 1919 was very small.

In general, 1917 and 1918 were years of higher temperatures in the water of the lake both at 18 m. and at 23.5 m. In 1919 and 1920 the temperatures were low; so that we have two years with large budgets and two years with small ones.

The autumn temperatures of the bottom water recorded in tab. 19 are as high as any recorded in the general temperature observations of those years, except in 1917, when the highest reading for mud observations was taken on Sept. 15 at 14.7°. In that year the mean temperature of the water at 18 m. for the last two weeks of September was 14.2°; for the first week in October it was 14.8°, with a maximum of 15.2°. This continued high temperature

for three weeks must have warmed the mud and the budget must have been correspondingly greater. In the general statement of results, therefore, the heat budget at 18 m. is reckoned as the same as at 23.5 m. The average of a larger series of years would probably show that it is somewhat, but not much greater.

Station II—23.5 Meters

Station II lies about two kilometers north of the University shore, toward the southern side of the main central basin of the lake. In this basin a maximum depth of 24.5 m. has been found and even slightly more. But no point has ever been definitely located where an average depth of more than 23.5 m. can regularly be found. There are evidently small hollows from 0.5 m. to 1 m. deep in the flat plain of the plafond, which measures some 5 km. across from northwest to southeast. This plain is therefore about as level as a natural surface of the earth is likely to be. The movement of temperature in the water at 23.5 m. follows the same course as that at 18 m., but at a level in summer from 0.5° to 0.7° lower, as shown in the general temperature observations of the lake (fig. 2). In the years reported in this paper the difference was practically the same, but the course of the curves at both levels was 1.0°

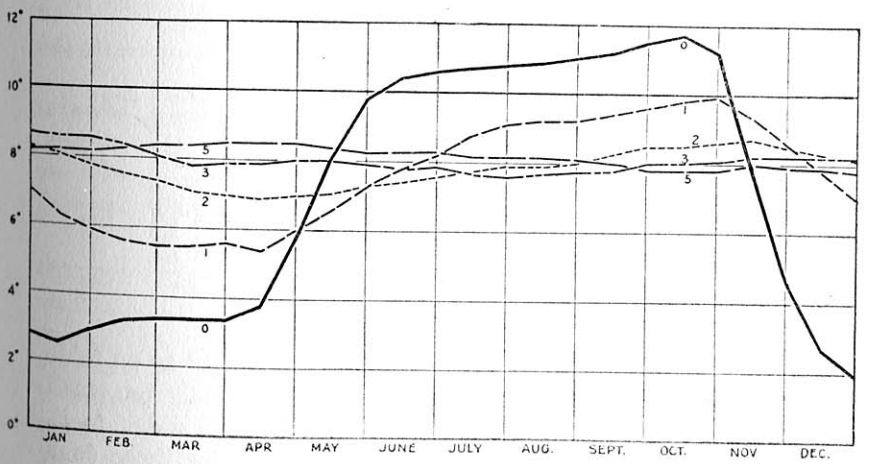

Fig. 9. Station II, 23.5 m. Annual temperature curves from table 9. Compare with figs. 3, 6, 8.

to 1.5° lower (figs. 8 and 9) than in those derived from the 16 year mean.

Table 9 shows the mean temperatures of the mud at Station II; fig. 9 shows the run of the curves derived from this table; fig. 10 shows the course of the detailed observations for 1918 and 1919; and fig. 11 gives a general picture of the curves from which may be derived the mean temperature of the five-meter stratum.

TABLE 9. *Station II—23.5 meters. Mean of observations, 1918, 1919 and 1920.*

Depth	Bot.	0.5	1.0	1.5	2.0	3.0	4.0	4.0	Mean temp.
Jan. 1	2.8	5.1	7.0	7.9	8.3	8.7	8.6	8.2	7.6
Feb. 1	3.0	4.5	5.9	7.1	7.8	8.6	8.7	8.2	7.3
Mar. 1	3.4	4.6	5.5	6.6	7.4	8.1	8.4	8.4	7.0
Apr. 1	3.4	4.8	5.6	6.5	7.0	7.9	8.3	8.5	7.1
May 1	5.8	5.9	6.0	6.4	7.0	8.0	8.3	8.5	7.2
June 1	9.8	8.3	7.3	7.1	7.3	7.9	8.3	8.3	7.7
July 1	10.5	9.4	8.2	7.9	7.6	7.8	8.2	8.3	8.2
Aug. 1	10.8	10.1	9.1	8.3	7.9	7.6	7.9	8.2	8.3
Sept. 1	11.0	10.2	9.2	8.5	8.0	7.8	7.9	8.1	8.5
Oct. 1	11.5	10.5	9.6	8.9	8.5	8.0	7.8	7.8	8.6
Nov. 1	11.2	10.6	9.9	9.3	8.6	8.1	7.8	7.8	8.8
Dec. 1	5.9	7.8	8.6	8.8	8.5	8.2	8.1	7.9	8.0

The maximum and minimum temperatures at Station II and the annual heat budgets are given in table 10, in the same form as for the other stations.

TABLE 10. *Station II—23.5 meters. Minimum and maximum temperatures of mud to the depth of five meters.*

Source	Minimum		Maximum		Difference	Budget in calories
	Date	Temperature	Date	Temperature		
Table 9	Mar. 1	7.0	Nov. 1	8.8	1.8	900
Single years						
1917	Mar. 10	7.1	Sept. 15	9.3	2.2	1100
1918	Mar. 23	6.2	Oct. 23	9.4	3.2	1600
1919	April 22	7.3	Oct. 11	8.7	1.4	700
1920	Mar. 20	6.4	Oct. 25	8.3	1.9	900
Mean		6.8		8.9	2.1	1050

The same comments may be made on table 10 as on table 8. The budget reported for 1917 is undoubtedly too small and observations in late October would have added to it. The true average annual budget as shown by these four years would rather exceed 1100 cal. than fall below that

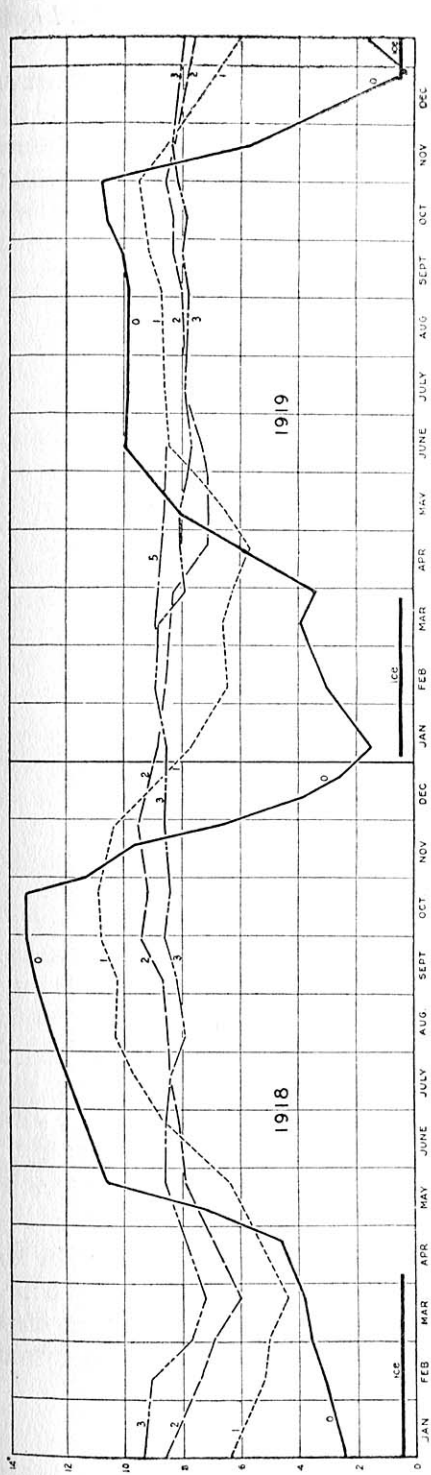

Fig. 10. Station II. Results of platting the single observations for 1918 and 1919. This shows one year when the temperature of the bottom water was high and one when it was low. 1917 was much like 1918, and 1920 like 1919; all years were below the mean for the lake, as shown in fig. 2. Observations in 1918 did not extend down to 5 m. and in 1919 the 5 m. line would be very close to that for 3 m. Station I would yield a very similar diagram, with the curves a fraction of a degree higher.

sum. Figure 10 shows the course of temperatures during the years 1918 and 1919, the years that gave the highest and the lowest annual budget. The diagram shows plainly the reason for the difference, which may be easily traced by following the curve for 1 m. In the winter of 1918 the

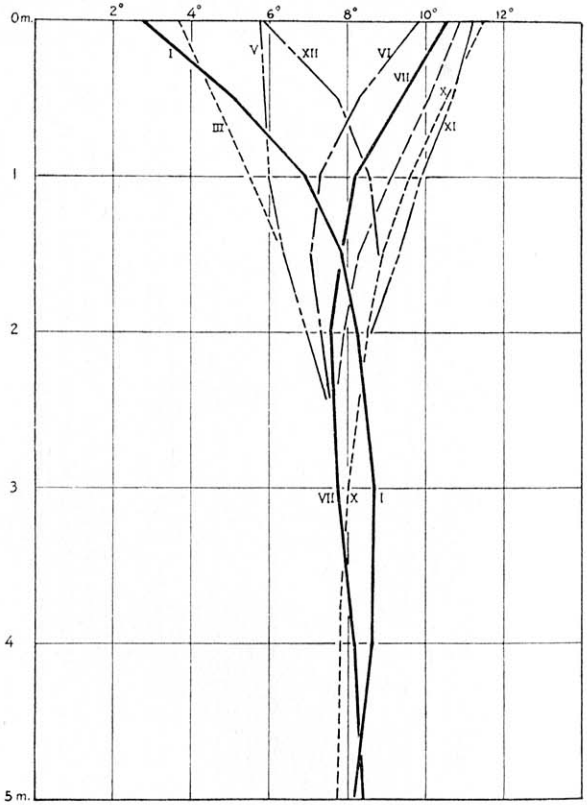

Fig. 11 Station II. Part of the curves for mean temperatures of mud to the depth of 5 m., from table 13, taken for the first of each month. The result is in general like that for the Eight-meter station, but with much smaller spread of temperature at the surface. Note that the lines for January and July cross twice, and see text p. 213.

temperature at this level falls to a low of 4.4° in March and rises to 10.9° in October, an oscillation of 6.5°. In 1919 the temperature is 5.7° in April and it rises only to 9.5° in October, a range of 3.8°. The curves for two and three

meters follow the general course of that for one meter; both fall rapidly in the winter of 1918 and remain almost stationary in the corresponding months of 1919. To the effect of this small loss in winter is to be added the influence of the low temperature of the bottom water in 1919; both resulting in a minimal heat budget for that year. The curve for five meters could not be given in 1918 as observations did not extend to that depth. In 1919, as in 1920, the difference between 3 m. and 5 m. was so small that both curves could not well be shown on the diagram. The movement of temperature in 1917 was much like that of 1918, and 1920 very closely resembled 1919.

There are shown in fig. 11 as many of the temperature curves for the first of each month as could be drawn without quite confusing the diagram. The annual oscillation of temperature in the bottom water is 9.1° (2.6° to 11.7°). This is to be compared with 21.3° at 8 m., as shown in table 2 and fig. 5. The oscillation at 1 m. is 4.4° (5.5°–9.9°), at 2 m. it is 1.8° (6.9°–8.7°); and at 3 m. it is 0.9° (7.8°–8.7°). This indicates that the oscillation loses about fifty percent of its amplitude in passing from meter to meter. The result is lower than at the Eight-meter station but about the same as at the other stations. It is probably due to the lower thermal diffusivity in the softer mud below the deep water. At the greater depths the percent of transmission rises, as it also did at the other stations, but the difference is more marked at Station II. The increase may be due to a greater thermal diffusivity in the deeper and denser mud or it may be accidental.

The temperature of the whole five meter stratum in July (table 9) shows a gain of 0.62° over that in January, or 310 calories per square centimeter of surface. This is made up, as fig. 11 shows, of a gain of about 470 cal. above 1.5 m.; a loss of 170 cal. between 1.5 m. and 4.7 m.; and a minute gain between 4.7 m. and 5 m. The mean temperature of the mud in June is about 0.22° or 110 cal. below that in December. This difference is made up by a loss of 112 cal. above 0.6 m. and of 30 cal. below 3.6 m.; and a gain of 252 cal. between 0.6 and 3.6 m. In both cases the curves representing mean temperatures cross twice, the crossing below 3 m. disclosing the influence of the temperature of

the lower strata of the lake bottom on the upper five-meter stratum.

VARIATION

The records of tables 17–20 (p. 231) offer immediate evidence that there may be a considerable amount of variation between the successive observations at the same station; an amount which can not be accounted for either by the accidents of observation or the changes which would naturally take place in the interval between observations. The observations are not numerous enough to permit any complete discussion of the subject of variation, but some general statements may be made regarding it and some special series of temperatures may be recorded, which were made for the purpose of ascertaining something about its possibilities.

Temperatures may differ in the mud at the same depth below its surface, in series taken on the same day, at the same depth of water, and at points closely adjacent. Successive observations at the same station may also differ to a quite unexpected amount. Such variations may be due to two kinds of causes, local and general. By local causes are understood those which differ from place to place in the same zone of the bottom and by general causes are understood those which affect in a similar way all parts of the same zone.

As local causes of variation in mud temperatures may be named (1) differences in thermal conductivity, and (2) differences in what may be called *drainage*. Under general causes may be named (3) the peculiarities in form of the annual heat wave which is propagated downward from the water through the mud.

1. The thermal conductivity of the mud differs from place to place. The conductivity of water is 0.00143; that of the organic substances found in the lake water is about the same; while that of the mineral constituents of the mud is much higher. The average conductivity of limestone, for instance, is about 0.005. The conductivity of the mud therefore differs according as it contains a larger or smaller percentage of water and organic matter. A small local hollow in the original surface of the slope of the lake would

permit the accumulation of soft organic matter causing a low conduction; while a small elevation would favor the removal of organic debris by lake currents, and the retention of heavier inorganic particles. Experience in driving the mud thermometer shows that there is considerable difference in the density of mud at points closely adjacent, and such differences must find some expression in conductivity. A very few determinations of conductivity were made in 1917 on mud from Lake Mendota by Professor L. R. Ingersoll of the University of Wisconsin. The conductivity of surface mud from a depth of 8 m. was 0.00147, practically the same as that of water; while mud from a near by place at the depth of 6 m. had a conductivity of 0.00200 and both its density and its specific heat were higher. In the case of the Eight-meter station the computations of Dr. March, based on the average temperatures for the surface of the mud and 1 m. and 2 m. below it, show an average thermal diffusivity of 0.00325. The actual change in temperature under such conditions as those of the mud depends on the thermal diffusivity, which equals the conductivity divided by the product of the density and the specific heat (see p. 226). Since all of these three factors must vary from point to point, it is evident that the mud at the same depth below the surface must show variations in temperature, resulting from a different conduction of the heat wave.

2. The second factor causing differences in temperature, especially on slopes, may be called *drainage* or the flow of water in winter. The water of the lake is always below the temperature of maximum density at the time of freezing and it remains below 4° during the winter except in the bottom of the deeper basins. As the water gains heat through contact with the bottom it becomes heavier and tends to flow down the slope and out into the lake. But as the increase in density is very small the slope of the bottom makes a decided difference in the flow, in proportion as the slope is steep or flat. At the regular stations the depth of the water increases from 5 m. to 15 m. in 200–300 m.; in Northeast Bay the distance between the same depths is about 2000 m.; so that the slope is much less and the opportunity for loss of heat is correspondingly reduced.

It is found that in spite of a loss of heat from mud to water in winter, the bottom water at shallower depths and on slopes warms very slowly, even when periods of several weeks are considered. An illustration is seen in the records of the Eight-meter station in 1920 (tab. 17) when the surface of the mud had the same temperature (0.8°) on Jan. 10 and on Feb. 28. In general, water at this depth and on a slope warms at the bottom only at the same rate as the whole mass of the overlying water; water at a temperature of 4° accumulates in the hollows of the larger basins of the lake while the upper water is still much colder.

The amount of the winter cooling has a considerable effect on the annual heat budget of the mud. During the open season the lake water at any given depth has a fairly uniform average temperature in all parts of the lake at any given date, since it is kept in circulation by the wind; so that the supply of heat during the season to the bottom at a depth of, say, 8 m. is much the same anywhere in the lake. The greater the loss of heat from the mud during the winter, the steeper is the thermal gradient between water and mud during the summer and the more rapid is the penetration of heat.

3. The general factor causing variations in mud temperatures is found in the variations in form of the annual heat wave which passes from the water into the mud. These become especially apparent because of the slowness with which the heat is propagated through the mud; the average "lag" at the Eight-meter station being about 31 days for each meter of depth (p. 199). The mean form of the heat wave at the surface of the lake is not far from that of a sine-curve, at least so far as the open season is concerned. But its form in any one year is very irregular, as warm and cold periods alternate, together with the accidents of wind and rain. Whenever these irregularities are great enough to escape the damping effect of transmission through the mud they are present at such a depth as 5 m. several months after their appearance on its surface, and they may be so large as to put the lower part of the series of temperatures on a given date, as recorded in the tables, out of line with the preceding or following series.

Such irregularities in the form of the heat wave would be even more effective in causing variations in records from stations in the deeper water, partly because the mean form of the wave itself has very marked bends (see figs. 8, 9), due to the summer lag in the propagation of the heat wave to the deeper water; partly also because the diffusivity of the mud in the deeper water is lower than that at the Eight-meter station and the lag must be correspondingly greater.

As has been said already, the temperature records are not numerous enough to allow any full application of these general statements. This is especially true of the last named cause. The records at Stations I and II for Oct. 11, 1919 (p. 231) show temperatures in the upper meter quite as would be expected from those before and after; but those in the deeper meters are much lower, and are about equally out of line at the two stations. This variation is probably, though not certainly, connected with the fact that the lower water during May of that year warmed very slowly, gaining only a little more than one degree while four degrees would be about the average. The striking difference between the observations on this date and the adjacent series can not be due to local causes since these would not have a like effect at stations separated by a kilometer.

Variations due to local causes are more easily identified and observations will be given to illustrate them. The first is found in series taken on May 28 and 29, 1919 at the Eight-meter and Twelve-meter stations on the south side of the lake and in Northeast Bay, about 4 km. distant.

TABLE 11. *Observations near the south side of the lake and in Northeast Bay.*

Depth	Eight meters		Twelve meters	
	South	Northeast	South	Northeast
Bot.	11.2	12.2	10.5	10.5
0.5	10.7	9.8	9.5	9.2
1.0	9.1	8.6	8.3	8.0
1.5	8.4	8.4	7.7	7.7
2.0	8.0	8.6	7.8	8.1
2.5	8.3	9.1	8.0	8.6
3.0	8.5	9.5	8.5	9.0
3.5	8.8	9.8	8.7	9.4
4.0	9.2	10.3	9.0	9.6
4.5	9.6	10.5	9.3	9.9
5.0	9.8	----------	9.5	----------

Note. The mud in Northeast Bay was so dense that observations stopped at 4.5 m.

The table shows that at the depth of 1.5 m. below the surface of the mud the temperature in the two localities was the same, but at greater depths the temperature in Northeast Bay was the higher. Such a difference at this time of the year is not due to greater gains of heat during the spring, but to smaller losses during the winter. Since the mud in Northeast Bay was denser than that on the south slope it is not probable that the conductivity was less; and since the slope of the bottom is much less drainage must have had some influence on the temperatures. However the influence of the different factors may be rated, it is plain that the heat budget in Northeast Bay will be smaller than at corresponding depths on the south slope of the lake bottom. The supply of heat from the water will be about the same and at the close of the warming season the temperatures in the deeper strata of the mud should be much alike.

Other series of readings were made at points closely adjacent and illustrate variations of temperature between such stations. On January 3, 4, and 5, 1917 four sets of observations were made near together in University Bay, and three sets, also near each other, in the bay between Picnic Point and Second Point.

TABLE 12. A. *Observations in University Bay, January 3 and 4, 1917.*

Depth	5.7 m.	7.8 m.	8.3 m.	9.0 m.
Bot.	0.8°	1.0°	0.8°	0.8°
0.5	3.4	3.8	2.3	2.3
1.0	5.9	6.2	5.3	6.0
1.5	8.8	7.4	7.4	8.4
2.0	10.4	9.7	9.2	10.4
2.5	----------	10.2	10.0	11.4

B. *Picnic Point Bay, January 5, 1917.*

Depth	5.8 m.	9.6 m.	12.3 m.
Bot.	0.8°	1.0°	1.0°
0.5	2.9	3.1	2.9
1.0	4.8	5.4	6.2
1.5	7.4	7.5	8.1
2.0	9.5	9.5	9.6
3.0	10.6	11.0	----------

On Jan. 9, 1919, two series were taken in University Bay at the depth of 12 m., one southwest and the other south-

east of the deepest water (20 m.). Temperatures were closely alike at 2.5 m. and below, but nearer the surface there was much difference.

TABLE 13. *Observations in University Bay, 12 m., Jan. 9, 1919.*

Depth	Southeast	Southwest
	12.5 m.	12.0 m.
Bot.	0.2°	0.3°
0.5	2.8	4.8
1.0	6.4	7.2
1.5	8.6	9.1
2.0	9.6	10.0
2.5	10.0	10.1
3.0	10.1	10.1
4.0	9.8	9.6
5.0	9.6	9.5

Four series were taken close together on the slope near the Eight-meter station on Feb. 5 and 6, 1920, with results as follows. Note especially the difference between I and III where the depth of water was the same.

TABLE 14. *Observations near south shore of lake, Feb. 5 and 6, 1920.*

Depth	I 8.0	II 9.0	III 8.0	IV 11.0
Bot.	0.8°	0.8°	0.7°	0.9°
0.5	2.7	2.4	2.7	2.4
1.0	4.9	4.4	3.1	4.2
1.5	6.4	6.3	5.7	6.2
2.0	7.8	8.1	7.1	7.8
2.5	8.7	8.8	8.2	8.5
3.0	9.4	9.6	9.1	----

These cases illustrate differences due to small changes in the place of observation, They may be attributed to differences in conductivity or in drainage or to both factors. Many of the differences appearing in the general tables are no doubt of similar origin. Observations at the same station were not taken at exactly the same spot on successive dates. This was especially true in summer when the launch had to be fixed by four anchors, sent out by boats in different directions, and the cables pulled taut in order to keep the boat immovable during the work.

There is an interesting series at the Eight-meter station on July 12, 1919 which may be compared with those before

and after it (see table 17 and fig. 4). A glance at fig. 4 will show that the temperature at the surface of the mud and at 1 m. rose as would be expected between June 13 and July 12; but that the temperature at 2 m. remained nearly stationary and there was a marked decline at 3 m. and 5 m.—a decline which puts these temperatures out of relation to the other observations of the season. Data are not at hand for reaching a decision on the question whether such a variation is due to general or local causes.

The general fact appears that observations of temperatures in the mud, either adjacent in location or successive in time do not show such uniformity of temperature as do those made in the water of the lake. This must be borne in mind in interpreting the observations here reported. They show well the general situation and the succession of events in the temperature exchanges of the mud. But a far greater number of observations must be at hand if minute accuracy of detail is desired, or if it is desired to determine local variations in conductivity and drainage. Still more complete data would be required to trace the effect of irregularities in the form of the heat wave, during the months of its passage through the mud to a depth of five meters.

Observations in Shallow Water

No regular station for series of temperatures was maintained in water shallower than 8 m. This was mainly due to the increasing hardness of the bottom at smaller depths and the consequent difficulty of driving the thermometer. The bottom at 5.7 m., reported in Table 12, had a hard crust which it was difficult to penetrate and while the mud was softer below it, the thermometer had to be driven from the first and the density of the mud stopped observations at 2 m. Altogether five series of temperatures were taken at depths of water between five and six meters. They showed that the conditions found at eight meters extend in toward shore at least as far as the mud goes; so that there need be no hesitation in assuming that the heat budgets of mud anywhere between shore and eight meters of water are substantially alike.

Very few readings were attempted in sandy bottoms and these were usually unsuccessful because the sand was too hard to permit the driving of the thermometer. On March 21, 1917 readings were made in 4.5 m. of water in sand to a depth of 1 m. At that depth the temperature of the bottom was 4.3° while that at the same depth in 8.4 m. of water was 4°. On March 30 a reading was attempted at the pier in about 1.5 m. of water. The surface temperature of the sand was 3.1° and one-half meter below the surface it was 4.4°. At this time the sun is warming the shallow water through the ice.

On the whole therefore, there is no reason to believe that any considerable error is made in assuming that the heat budget of the bottom deposits at eight meters of water is also that of the region between eight meters and shore. Probably the estimate is a little too low, as might be inferred from the curves for the mean temperature of water at the surface of the lake and at eight meters (fig. 2); and the conductivity of sand bottom differs somewhat from that of the mud. But the areas involved are not great enough to cause serious errors in the estimates of the total heat budget of the lake bottom. The matter is further discussed on p. 223.

The Total Heat Budget

The general course of the mean temperatures of the mud to the depth of 5 m. may be followed through the year from the several tables, and for three of the stations they are shown in fig. 12. The lowest temperatures are found in March and April. At the Eight-meter station the temperature rises rapidly through April, May, June, and July; but the rise is checked abruptly in August and the maximum was found about September 1. The difference in temperatures between July 15 and Oct. 1 is however, small (see p. 194). At the Twelve-meter station the most rapid rise is during May and June, with a slow continuance, going on steadily until the maximum is reached in October. At station II May and June are also the period of rapid rise; but the later increase is very slow, culminating in November. In all cases the decline after November 1 is rapid, corresponding to the cooling of the lake. This decline is checked

early in December, since a stable stratification is reestablished after the temperature of 4° is passed, and thereafter all cooling processes are slowed up. The curve of mean temperature for Station I (18 m.) is slightly above that for Station II, but is so near to it that both can not well be shown in the same diagram.

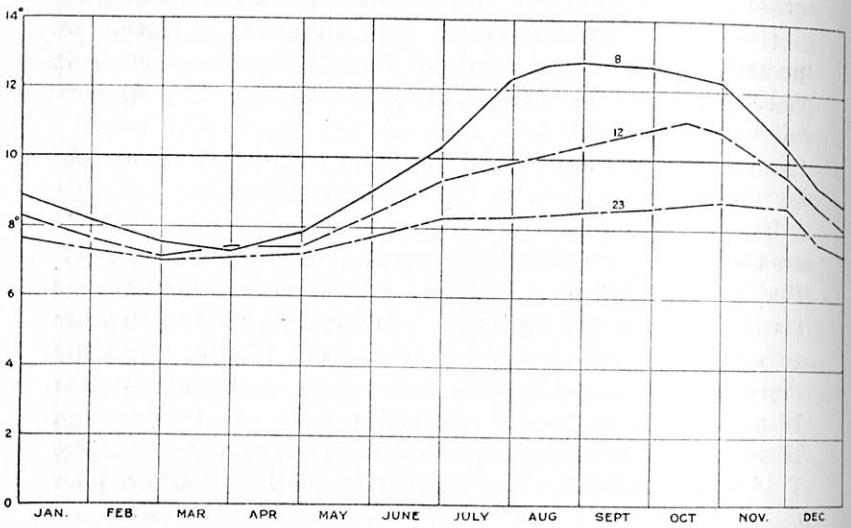

Fig. 12. Diagram showing course of mean annual temperature for the 5 m. stratum of mud at the Eight-meter and Twelve-meter stations and at Station II.

The approximate total heat budget of the lake bottom may be determined from the data here reported. The area of Lake Mendota at various depths is as follows:

TABLE 15. *Percentile area of Lake Mendota at depths named.*

Depth, meters	Area, percent
0	100
3	81
8	67
12	54
18	30
20	16
23.5	--

We may assume as round numbers for the heat budgets at the several depths of observation: at 8 m., 3000 cal.; at 12 m., 2200 cal.; at 18 m., 1100 cal. at 23.5 m., 1100 cal. These figures are rather below than above the true results, for in no year of the study was the temperature of the water at any depth above the mean derived from 16 years observation, and in most cases it was below the mean.

If these results are platted on coordinate paper at the proper place, connected and the area measured which is enclosed by the curve, the result is a mean annual budget for the lake bottom of 2035 calories per square centimeter of its surface. This is in round numbers 2000 cal., which will be taken as the result.

This result involves the following assumptions. (1) The area of the bottom of the lake is assumed to be the same as that of the surface. (2) The line joining the budgets for 8 m., 12 m., and 18.5 m. is nearly straight, and the budgets are assumed to decrease in the same ratio. Probably there is a more rapid decline, for instance, between 8 m. and 10 m. than between 10 m. and 12 m. (3) The budget from 18 m. to the center of the lake is assumed to be uniform. This is probably correct in general. (4) No allowance is made for difference in specific heat between water and mud. There is certainly a difference, but we know very little about the specific heat of the mud and can not discuss the matter quantitatively at present. (5) The budget of the area between the 8 m. station and the margin is assumed to be the same as that at 8 m. On this subject a little more may be said.

The observations made in water between 5 m. and 6 m. deep (p. 220) show that there is little change between the Eight-meter station and the smaller depth. Professor March has computed the effect on the bottom of the mean annual heat wave for the depth of 3 m. in the lake (p. 228), assuming a diffusivity the same as at 8 m. The annual heat budget works out as 3150 cal. while that at 8 m. is 2850 cal. on the same basis. No doubt the diffusivity at 3 m. is greater as there is more sand in the bottom deposits, but there can be no serious error in assuming an average budget of 3000 cal. for this region.

The average annual heat budget for the water of Lake

Mendota has been determined as 23,000—24,000 cal. To this sum must now be added about 2000 cal. for the heat which goes into the bottom. Thus 8 percent or 9 percent are added to the budget for the water, and the average heat budget of the lake is 25,000—26,000 cal. for each square centimeter of the surface.

The losses of heat from the mud to the water in winter may be computed in the same way. The amount of heat lost by the mud from December 15 to April 1, which was approximately the ice period for the years in question, was as follows: 8 m.; 1040 cal.; 12 m., 600 cal.; 18 m., 300 cal.; 23.5 m., 300 cal. The result for 18 m. is probably somewhat too small, owing to the position of Station I in the smaller or Picnic Point basin. If it had been on the slope of the main basin of the lake the loss in winter would have been greater than that in the deeper water. If these figures are platted and the intercepted curve measured, the mean result is a winter loss by the mud of about 650 cal. per sq. cm. of surface.

The result of 12 years study of winter temperatures in the water shows that its mean temperature immediately after freezing is 0.6°; immediately after the disappearance of the ice it is 2.6°, indicating a gain of 2° during the ice period. The mean depth of the lake is 12.1 m. so that a gain of 2° corresponds to about 2400 cal. per sq. cm. of surface. The mean ice period is from December 18 to April 7, or 110 days. It thus appears that the passage of heat from the bottom to the water of the lake during the ice period will account for about one-fourth of the total gain. The remainder must be assigned in small part to inflowing water, and mainly to the penetration of the ice by the sun's rays, which are increasingly effective as winter passes on into spring.

MATHEMATICAL DISCUSSION

Analysis of Data from Eight-meter Station

The tables of mean temperatures from the several stations and the annual temperature curves derived from them show that smooth average results can not be derived from a three year mean. Such a result is to be expected. Recourse

was therefore had to a mathematical analysis of data from the Eight-meter station in order to eliminate so far as possible the accidental and occasional elements which effect variations in the heat budget of the mud, and to ascertain the effect of the permanent factors in their average strength.

The annual heat wave was converted into a form which could be handled mathematically. The attempt was first made to fit a sine-curve to the data for the temperature at the bottom of the water as given in table 2: but the adjustment between sine-curve and data, while fairly close, was not thought accurate enough. The curve was then analyzed into its Fourier components by the 24-ordinate method[1], in which the ordinates were the data given in the first column of temperatures in table 2. It was found that all components after the first three were negligible. From this analysis the following equation resulted, in which Θ_0, the temperature at the surface of the mud, is represented as a function of the time.

$$(1) \qquad \Theta_0 = 9.94 + 10.70 \sin (\tau - 129^\circ\ 40') + 1.30 \sin (2\tau - 1^\circ\ 40') + 0.69 \sin (3\tau - 65^\circ\ 47').$$

In this expression, τ, measured in degrees, is equal to the number of the day in question, counting from the beginning of the year. To simplify the computation and plotting of the curves, the year was taken to be composed of twelve months of thirty days each. This procedure is sufficiently accurate for the purposes of this investigation.

The temperature at any depth below the surface of the mud is to be calculated from the temperature of the surface as expressed by equation (1) which will be written in the form

$$(2) \qquad \Theta_0 = C_0 + \sum_{n=1}^{3} C_n \sin(n\ \omega\ t - \gamma_n).$$

From this it is known[2] that the temperature at any depth x is given by the equation (3).

1. See e. g. Whittaker and Robinson, The Calculus of Observations, p. 273.

2. See e. g. H. S. Carslaw, The Conduction of Heat, p. 50.

$$(3) \qquad \Theta = C_0 + \sum_{n=1}^{3} C_n \exp(-p_n x)\sin(n\ \omega\ t\ -\gamma_n - p_n x).$$

In these formulas C_0 is the mean annual temperature, which is assumed to be independent of the depth. This assumption is not necessary but it is suggested and confirmed by the data for this station. The product ω t, measured in radians, replaces τ of equation (1); t is measured in seconds; and $\omega = 2\pi/T$, where T is the number of seconds in one year. The expression $\exp(-p_n x)$ is the damping effect, i. e., the percentage of the oscillation at the surface of the mud which will remain at depth x; $p_n = (n\ \omega/2)^{1/2}/h$, in which $h = (k/c\ \rho)^{1/2}$, where k is the conductivity, c the specific heat, and ρ the density of the mud; $p_n x$ is the phase shift, which comes out as the lag in the transmission of heat from meter to meter.

From the series of equation (1) the values of the temperatures at the surface of the mud were computed, with the result seen in table 15 and fig. 13. These are to be compared with the corresponding data for the Eight-meter station (table 2, fig. 3) and the fit will be found entirely satisfactory.

In deriving the temperatures at depths below the surface of the mud, the constant h, the square root of the diffusivity, was first determined. The mean observed temperatures at the depths of 1 m. and 2 m. respectively were analyzed and found to be represented by the series:

$$(4) \qquad \Theta_1 = 10.10 + 6.13 \sin(\tau - 154°8') + 0.72 \sin(2\tau - 27°59') + 0.32 \sin(3\tau - 138°6').$$

$$(5 \qquad \Theta_2 = 10.01 + 3.58 \sin(\tau - 185°50') + 0.40 \sin(2\tau - 96°11') + 0.20 \sin(3\tau - 179°53').$$

From the damping factor $\exp(-p_1 x)$ (see equation (3)) obtained for each of these depths by comparing (4) and (5) with (1), the constant h was computed and found to have the values 0.0557 and 0.0582 from (4) and (5) respectively. The mean of these values, 0.057, was adopted as the value of h, the units of length and time being one centimeter and one second. The corresponding value of the diffusivity h^2 is 0.00325.

With this value of h the temperatures at depths of 1, 2, 3, 4, and 5 meters were computed by (3), from the surface temperatures given by (1). The results are given in table 15 and fig. 13. They are in satisfactory agreement with the mean observed temperatures at the station.

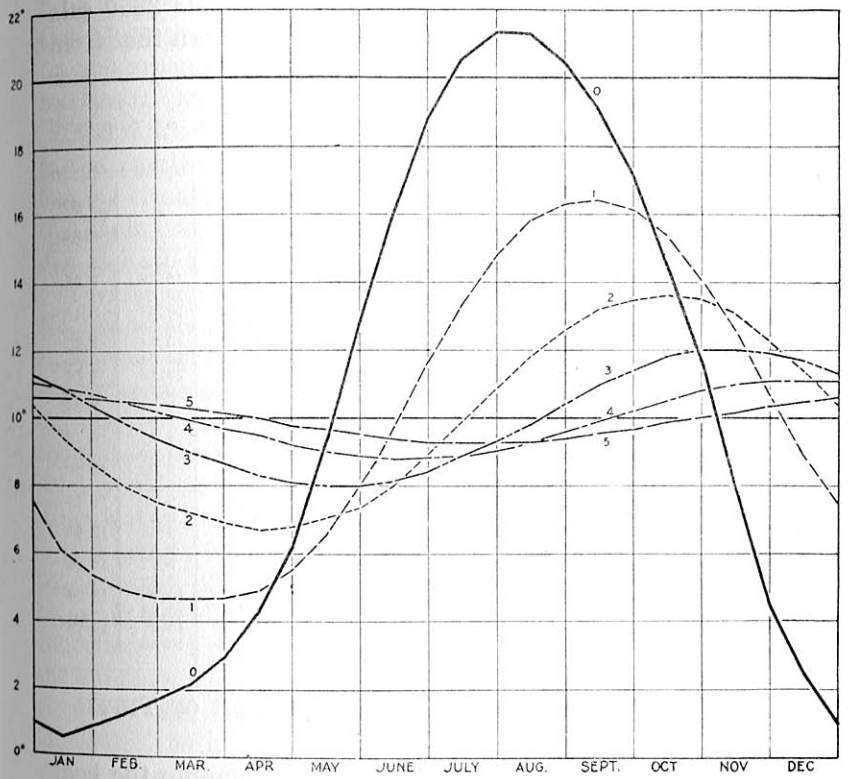

Fig. 13. Course of annual temperatures in the mud, based on a temperature of bottom water shown in curve marked 0. This is a smoothed curve close to that observed at 8 m. See fig. 3. The heat conductivity of the mud to be assumed was fitted to the results shown in table 2. (See text, p. 194). The diagram represents approximately the movement of temperatures at the Eight-meter station, as they would appear if observations were more numerous and extended over more years, so as to yield smooth curves.

It appears that the damping effect is such that the temperature oscillation at 1 m. is about 57 per cent of that at the surface and the same law holds to the depth of 5 m. The lag for an increase of depth of one meter is a little more

than 30 days. This is also in agreement with the results from fig. 3. It takes, therefore, more than five months for temperature changes at the surface of the mud to register their effect at the depth of five meters.

The annual heat budget may be computed from the results of this analysis as given in table 16. It equals 2850 cal., about the same as that from table 2 and less than that from the mean of the several years as shown in table 3.

TABLE 16. *Temperatures at the Eight-meter station, as computed from the mean annual heat curve.*

Depth	0	1	2	3	4	5	Mean Temperature
Jan. 1	1.03	7.46	10.48	11.29	11.06	10.56	9.24
Jan. 15	0.59	6.14	9.53	10.85	10.93	10.60	
Feb. 1	0.78	5.36	8.71	10.36	10.74	10.58	8.14
Feb. 15	1.22	4.92	7.95	9.87	10.50	10.50	
Mar. 1	1.71	4.71	7.54	9.40	10.23	10.44	7.50
Mar. 15	2.14	4.66	7.17	8.98	9.96	10.31	
April 1	2.87	4.68	6.91	8.61	9.69	10.16	7.24
April 15	4.22	4.93	6.71	8.32	9.44	9.99	
May 1	6.24	5.52	6.71	8.12	9.21	9.82	7.36
May 15	9.39	6.53	6.97	8.02	9.03	9.66	
June 1	12.81	8.00	7.30	8.03	8.89	9.51	8.46
June 15	16.09	9.78	7.97	8.16	8.82	9.40	
July 1	18.77	11.66	8.86	8.43	8.82	9.32	10.02
July 15	20.53	13.40	9.87	8.81	8.89	9.28	
Aug. 1	21.32	14.80	10.89	9.28	9.04	9.30	11.60
Aug. 15	21.26	15.76	11.75	9.83	9.26	9.35	
Sept. 1	20.47	16.29	12.60	10.40	9.45	9.44	12.58
Sept. 15	19.12	16.38	13.17	10.96	9.86	9.57	
Oct. 1	17.21	16.06	13.51	11.43	10.19	9.72	12.94
Oct. 15	14.42	15.29	13.61	11.78	10.50	9.89	
Nov. 1	11.42	14.10	13.45	12.00	10.77	10.06	12.20
Nov. 15	7.89	12.55	13.09	12.04	10.97	10.22	
Dec. 1	4.79	10.74	12.32	11.93	11.09	10.37	10.76
Dec. 15	2.41	8.94	11.45	11.66	11.12	10.48	

ANALYSIS OF RESULTS AT A WATER DEPTH OF THREE METERS

The same formulas have been used to compute the heat budget which would be present in the bottom deposits at a depth of three meters. The data for the mean annual temperature oscillation at this depth were treated by the same methods as those from the eight-meter station. The temperature of the surface of the mud is expressed by the following series:

$$(6)\quad \Theta_o = 10.78 + 12.12 \sin(\tau - 124°00') + 1.47 \sin(2\tau - 339°43') + 1.11 \sin(3\tau - 42°28').$$

From this expression the temperatures at 1, 2, 3, 4, and 5 meters were computed by (3), using the value of h de-

termined at the eight-meter station. This value is probably too small since there is much more sand in the bottom deposits at 3 m. than at 8 m.; but h varies only as the square root of the conductivity divided by the product of the density and specific heat. The result is to show an annual heat budget of 3150 cal. It thus appears that the assumption of an average annual heat budget of 3000 cal. (p. 223) for the area between the margin of the lake and 8 m. must be close to the facts.

Computations similar to the foregoing might be applied to the results from stations at a greater depth of water; but the situation at these stations is more complicated. At the Eight-meter station there is no evidence of any effect due to heat transmitted upward from the ground into the five meter stratum of mud which is under consideration. No doubt such an effect is present, but it is necessarily small and a determination of its value would require a longer series of years or a greater number of observations or both. Of course, if the progress of the annual heat wave could be followed into the ground for a greater distance, a level would be reached where the effect of ground temperatures would be obvious.

If a lake is imagined whose deepest water has a constant temperature of 4° the temperature in the bottom deposits below it would rise regularly with increasing depth. This condition is far from that found in Lake Mendota, where the summer temperature of the deepest water is rarely less than 10° and may rise in favorable seasons above 15°. However, the annual temperature oscillation in the deeper water does not completely obscure the effects of ground temperatures, especially in the lower part of the five meter stratum. This fact is evident for instance, in the data from Station II (p. 213). But the observations are neither sufficiently numerous nor sufficiently regular to warrant the construction of a heat curve extending downward into the bottom deposits. No attempt has therefore been made to carry the mathematical analysis over to these stations.

SUMMARY

1. The temperature of the mud bottom of Lake Mendota was observed at four stations located in water 8 m., 12 m., 18 m., and 23.5 m. deep. Temperatures were read every half meter to a depth of 5 m. A total of 207 series was taken between March, 1916 and February, 1921; 181 of these were taken after Dec. 1, 1917.

2. Tables of temperatures are given in tables 2, 5, 7, and 9, based on the means of the observations of 1918, 1919, and 1920. Detailed records are given in an appendix. Maximum and minimum temperatures and the heat budget for each station and each year are given in tables 3, 6, 8, and 10.

3. The changes of temperature in the mud are the result of the heat-wave in the water immediately above it. This wave has a period of one year and an amplitude which differs with the depth of the water. The heat is transmitted through the mud by conduction.

4. At the Eight-meter station the amplitude of the annual heat-wave during the years of observation was 21.5° (0.8°—22.3°). The transmission of the heat was about 57% per meter. The lag in the transmission was about 31 days, or one month, per meter. At the other stations, where the water was deeper, the amplitude of the heat-wave was smaller (see tables); and, since the mud was softer, the percentage of transmission was less (about 50%) and the lag was greater (see figs. 8, 9). In all cases the transmission in the deeper part of the 5 m. stratum seemed to be greater than near the surface; but this result was not clearly shown, and in discussing the data from the Eight-meter station, the transmission in the upper two meters of the stratum was assumed to hold throughout.

The large amount of lag in the transmission complicates the movement of the changes of temperature in the mud. On almost any date part of the 5 m. stratum is gaining heat and part is losing.

6. The annual heat-budget is taken as the difference between the minimum and the maximum mean temperatures, which were found in the 5 m. stratum during the year. At the Eight-meter station it was 5.9° (7.1°—13.0°),

as derived from the mean of the three years. This is equal to 2950 cal. per sq. cm. In a similar way the annual heat-budget at the Twelve-meter station was found to be about 2200 cal.; at Station I, 1100 cal.; at Station II, 1100 cal. The mean annual heat-budget for the entire lake bottom is about 2000 cal. per sq. cm. In these computations the specific heat of the mud is assumed to be the same as that of the water.

7. There is considerable variation in the rate of movement of heat through the mud, Two causes are assigned for local variations: conductivity and "drainage". A cause for general variations is seen in irregularities in the form of the annual heat-wave due to variations in the rate of warming or cooling of the lake. Since conduction in the mud is slow, these irregularities reach the deeper layers of the 5 m. stratum several months after they appear at its surface.

8. The data from the Eight-meter station are discussed mathematically, yielding a table of temperatures (table 16) and a diagram (fig. 13) in which local variations and irregularities are eliminated. The result is a heat-budget of 2850 cal. The mean annual heat-wave in the water of the lake at the depth of 3 m. below its surface was taken as the basis for computing the heat exchanges in the lake bottom below water 3 m. deep. The annual heat-budget resulting from this computation is 3150 cal. per sq. cm.

APPENDIX

Records of Observations at the several Stations

Table 17. *Observations at the Eight-meter Station.*

Depth	1917 Jan. 4	Feb. 20	Jan. 30	Feb. 22	Mar. 1	Mar. 21	Aug. 20	Sept. 15	Dec. 21	1918 Feb. 6	Feb. 23	Mar. 9	Mar. 23	May 25	June 15	July 13	Aug. 7	Sept. 13	Oct. 4	Oct. 22	Nov. 6	Nov. 26
Bot.	1.0	1.0	1.4	1.2	1.5	1.9	23.6	17.8	0.3	1.1	1.3	1.1	1.5	13.0	18.2	19.6	23.6	18.1	14.7	13.5	10.6	6.4
0.5	3.8	3.0	3.1	2.8	2.6	2.4	21.7	16.9	3.4	4.1	2.4	2.3	2.6	10.1	14.7	17.7	19.4	17.8	15.1	14.5	11.9	8.4
1	6.2	6.0	5.8	4.7	4.4	4.0	17.2	16.5	6.4	6.2	3.5	3.8	3.9	8.6	12.2	15.1	16.6	16.8	14.9	14.4	13.0	11.3
1.5	8.4	8.3	7.5	6.6	6.3	5.6	14.8	14.5	8.6	7.8	5.7	5.2	5.4	7.6	10.1	12.7	13.3	15.1	13.6	14.0	13.5	12.1
2	9.7	9.4	8.9	8.0	7.7	6.8	12.4	12.7	10.4	8.5	7.3	6.3	6.7	7.8	9.0	10.5	12.0	13.6	12.7	13.6	13.5	12.4
2.5	10.7	10.2	9.8	9.2	9.0	7.8	11.7	11.7	11.2	10.0	8.5	7.3	7.6	8.0	8.5	9.9	10.8	12.4	11.9	13.1	13.1	12.3
3	11.2	11.0	10.6	10.2	9.6	8.6	10.9	10.7	11.6	10.8	9.2	8.1	8.6	8.5	8.7	9.5	10.0	11.5	11.3	12.1	12.2	11.6
3.5	11.6	11.4	11.2	10.8	10.4	9.5	------	------	12.0	11.2	9.8	8.8	9.2	8.9	8.7	9.3	9.4	10.9	10.9	11.5	11.6	11.5
4	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------

	1919 Jan. 6	Jan. 21	Feb. 12	Mar. 10	Apr. 18	May 9	May 28	June 14	July 12	July 25	Sept. 4	Sept. 23	Oct. 13	Nov. 3	Nov. 17	Dec. 12	1920 Jan. 10	Feb. 5	Feb. 28	Mar. 20	Apr. 10	May 1	May 21	June 15	July 17	Aug. 14	Sept. 13	Sept. 17	Oct. 29	Nov. 17	1921 Jan. 14	Feb. 22
Bot.	0.2	0.8	2.3	2.7	5.3	8.4	11.2	12.0	19.3	23.9	20.3	19.7	14.7	10.4	6.2	0.25	0.8	0.8	0.8	1.9	3.1	5.9	13.8	18.6	20.1	22.0	17.8	19.9	14.0	6.0	1.3	2.6
0.5	4.8	3.9	4.3	4.4	5.4	7.9	10.7	10.1	12.9	23.2	18.0	17.2	15.2	11.0	8.5	5.4	3.0	2.7	2.8	3.2	4.2	5.8	9.6	13.4	18.7	19.5	16.8	18.5	14.8	6.8	3.4	4.1
1.0	6.6	5.8	5.8	5.8	5.8	7.1	9.1	9.1	10.9	17.4	16.8	15.8	15.5	12.8	11.3	7.6	5.2	4.6	4.5	4.3	5.2	5.7	8.1	10.5	15.6	16.7	15.1	16.7	15.0	10.5	5.9	5.6
1.5	8.6	7.7	7.3	7.0	6.5	7.0	8.4	8.7	9.3	13.3	14.9	13.9	14.8	13.6	12.5	9.5	7.0	6.5	6.0	5.6	6.1	6.2	7.4	9.0	12.3	14.4	13.6	15.1	14.6	11.4	8.0	6.8
2	9.9	8.9	8.4	7.9	7.3	7.3	8.0	8.6	8.5	11.5	13.2	12.3	13.8	13.7	12.8	10.9	8.2	7.8	7.3	6.7	6.8	7.0	7.1	8.0	10.1	13.5	12.4	13.9	13.5	13.0	9.1	7.7
2.5	10.5	9.8	9.3	8.6	8.0	7.8	8.3	8.7	8.3	10.3	11.7	11.1	13.0	13.3	12.5	11.5	9.4	8.7	8.2	7.6	7.6	7.7	7.5	7.8	9.2	11.1	11.5	12.5	12.8	12.8	10.1	8.5
3	11.0	10.4	9.8	9.4	8.7	8.3	8.5	9.0	8.1	9.5	10.9	10.6	12.1	12.6	12.1	11.7	10.1	9.4	9.1	8.5	8.4	8.3	7.9	7.9	9.0	10.1	11.0	11.3	11.7	12.4	10.4	9.2
3.5	11.1	10.8	10.2	9.8	9.2	8.9	8.8	9.4	8.0	9.2	10.2	9.9	11.3	12.0	11.6	11.5	10.5	10.0	9.7	9.2	9.0	8.9	8.5	8.0	9.0	9.4	10.7	10.5	11.0	11.8	10.5	9.5
4	11.1	10.8	10.5	10.2	9.5	9.5	9.2	9.6	8.3	9.0	9.9	9.5	10.6	11.3	11.1	11.2	10.5	10.4	10.2	9.7	9.7	9.4	8.8	8.6	9.1	9.7	10.2	10.0	11.6	11.2	10.6	9.8
4.5	------	10.8	10.6	10.4	9.9	------	9.6	9.8	8.5	9.0	9.7	9.1	10.0	10.8	11.0	11.2	10.4	10.6	10.4	10.0	9.9	9.8	9.1	8.9	9.2	------	------	9.8	10.5	10.7	10.6	------
5	11.2	10.8	10.6	10.8	10.2	------	9.8	10.2	8.7	9.2	9.6	9.0	------	10.7	10.9	11.1	10.4	------	10.6	10.2	10.1	9.9	------	9.1	------	------	------	9.5	------	------	------	------

TABLE 18. *Observations at the Twelve-meter Station.*

Depth	1917 Jan. 5	Dec. 22	1918 Feb. 6	Mar. 2	Mar. 23	May 18	June 15	July 13	Aug. 7	Sept. 13	Oct. 4	Oct. 23	Nov. 7	Nov. 26	1919 Jan. 9	Jan. 21	Feb. 12	Mar. 10	Apr. 18	May 13	May 29	June 14	July 12	July 25	Sept. 4	Sept. 23	Oct. 13	Nov. 3	Nov. 17	Dec. 12
Bot.	1.0	0.7	1.3	1.3	2.2	11.3	14.3	17.2	18.0	18.1	14.7	13.5	10.8	6.4	0.2	0.9	2.7	2.6	5.4	9.0	10.5	11.2	15.6	12.5	13.5	18.8	14.6	10.1	6.2	0.2
0.5	3.8	3.4	2.7	2.6	3.2	8.5	11.6	15.1	14.8	16.7	14.4	13.7	13.0	8.7	2.5	4.0	4.2	4.4	5.7	7.6	9.2	10.6	11.4	11.2	12.7	15.1	13.9	10.2	9.3	4.5
1.0	6.2	6.6	4.4	4.2	3.6	7.2	10.0	13.5	12.8	15.1	12.9	13.4	13.5	10.1	6.4	6.5	5.7	6.0	6.1	7.0	8.0	9.4	9.8	9.9	11.1	12.6	13.8	11.5	10.5	6.8
1.5	8.1	8.7	6.8	5.4	4.6	6.7	8.9	11.5	11.4	13.6	12.1	12.9	12.9	11.3	8.6	8.3	7.5	7.2	6.6	6.9	7.7	8.8	9.3	9.3	10.3	10.6	11.6	11.9	11.1	8.8
2.0	9.6	10.0	7.2	6.6	5.7	6.9	8.5	9.7	10.2	12.0	11.1	12.0	12.1	11.7	9.6	9.5	8.6	8.5	7.1	7.4	8.1	8.6	8.9	9.0	9.8	9.8	10.5	11.3	10.9	9.6
2.5	10.3	10.4	8.4	7.2	6.6	7.5	8.2	9.4	9.3	11.3	10.3	11.2	11.4	11.5	10.0	10.0	9.3	9.3	7.7	7.6	8.6	8.5	8.8	8.7	9.4	9.4	9.8	10.6	10.6	9.8
3.0	10.6	10.8	9.4	7.8	7.5	7.9	8.4	9.3	8.8	10.4	9.8	10.5	10.8	11.2	10.1	10.1	9.7	9.8	8.4	8.4	9.0	8.7	8.8	8.7	9.2	9.0	9.2	10.3	10.2	9.7
3.5	10.6	10.9	10.0	8.4	8.4	8.4	------	------	8.5	10.0	------	10.2	10.4	11.0	10.0	10.0	9.9	10.0	8.8	8.8	9.4	8.9	8.8	8.7	9.1	8.8	8.7	10.2	9.9	9.6
4.0	------	------	------	------	------	------	------	------	------	------	------	------	------	------	9.8	9.9	9.9	10.1	9.2	9.1	9.6	------	8.9	8.8	9.4	8.6	8.8	------	9.8	9.5
4.5	------	------	------	------	------	------	------	------	------	------	------	------	------	------	9.7	9.8	9.9	------	9.5	9.2	9.8	------	9.1	8.9	------	8.7	------	------	9.7	9.4
5.0	------	------	------	------	------	------	------	------	------	------	------	------	------	------	9.6	9.8	9.9	------	9.7	9.2	------	------	9.3	9.0	------	8.9	------	------	9.6	9.4

Depth	1920 Jan. 10	Feb. 5	Feb. 28	Mar. 20	Apr. 10	May 11	May 29	June 15	July 17	Aug. 16	Sept. 17	Oct. 29	Nov. 17	1921 Jan. 14	Feb. 22
Bot.	0.8	0.8	0.8	1.5	2.8	5.7	10.5	13.0	14.0	12.6	16.0	14.3	6.5	1.7	2.6
0.5	2.5	2.6	3.4	3.6	3.3	5.3	9.4	10.5	12.3	12.7	14.2	14.3	9.9	3.7	3.7
1.0	4.5	4.8	4.4	5.0	4.4	5.2	7.7	9.0	10.7	11.4	12.0	13.4	11.2	6.1	4.9
1.5	6.6	6.5	5.8	6.4	5.5	5.5	6.8	7.8	9.0	10.3	10.7	12.1	11.4	7.7	6.4
2.0	8.0	8.1	6.8	7.5	6.7	5.9	6.7	7.5	8.4	9.2	9.9	11.0	11.3	8.4	7.4
2.5	8.9	8.5	7.7	8.1	7.5	6.6	7.0	7.5	8.2	8.8	9.4	10.4	10.9	9.0	8.1
3.0	9.1	9.0	8.4	8.7	8.1	7.2	7.6	7.8	8.1	8.4	9.1	9.8	10.4	9.1	8.6
3.5	9.3	9.3	8.8	9.1	8.6	7.9	7.9	8.0	8.4	8.3	8.8	9.5	9.9	9.0	8.8
4.0	9.3	9.4	9.1	9.3	8.8	8.4	8.4	8.1	8.5	8.5	8.6	9.4	------	9.0	8.9
4.5	9.3	9.5	9.3	9.5	------	8.8	------	------	------	------	8.5	------	------	------	9.0
5.0	9.4	9.6	9.5	------	------	------	------	------	------	------	8.5	------	------	------	------

TABLE 19. *Observations at Station I, 18 meters.*

Depth	1917 Jan 3	Feb. 16	Mar. 24	Sept. 15	Dec. 21	1918 Feb. 9	Feb. 23	Mar. 23	May 18	June 26	July 13	Aug. 7	Sept. 7	Sept. 28	Oct. 22	Nov. 25	1919 Jan. 7	Jan. 21	Feb. 8	Mar. 11	Apr. 19	May 8	May 27	June 13	July 18	Sept. 4	Sept. 23	Oct. 11	Nov. 19
Bot.	2.3	1.8	2.7	14.7	2.5	2.0	3.9	3.9	9.6	11.9	12.8	13.4	14.3	14.6	13.5	6.4	1.7	2.7	3.0	3.3	5.5	8.1	9.3	9.4	10.4	10.5	10.3	13.3	5.7
0.5	4.2	4.4	3.6	12.1	3.3	3.2	4.8	4.4	8.2	11.0	1.3	12.3	12.4	12.5	12.8	7.9	5.0	6.2	6.7	5.7	5.5	7.6	8.2	8.3	9.8	9.7	9.2	11.0	7.6
1.0	6.7	5.8	5.0	10.9	6.4	4.8	6.1	5.5	7.0	9.5	10.6	11.7	10.6	11.2	11.7	10.0	7.0	7.8	7.6	7.2	5.9	6.8	7.6	7.6	8.8	9.2	8.8	8.8	8.8
1.5	8.3	7.0	6.2	9.6	7.9	6.1	7.2	6.4	7.2	8.4	8.7	11.0	9.7	10.3	10.7	10.3	8.3	8.7	8.2	8.0	6.7	6.7	7.4	7.4	8.3	8.6	8.3	8.0	9.0
2.0	9.2	7.8	7.0	8.8	9.0	7.5	8.1	7.2	7.7	7.9	8.4	10.3	9.0	9.6	9.8	10.7	8.9	9.0	8.5	8.5	7.4	7.2	7.7	7.6	7.9	8.3	8.0	7.8	8.8
2.5	9.6	8.6	7.8	8.5	9.6	8.5	8.8	7.8	8.2	8.4	8.3	9.0	8.7	9.0	9.3	9.7	9.1	9.2	9.2	8.8	8.1	7.7	8.0	7.9	7.8	8.2	7.9	7.6	8.6
3.0	9.7	9.0	8.2	8.2	9.6	9.4	9.2	8.3	8.5	8.5	8.2	8.4	8.6	8.8	8.9	9.4	9.2	9.1	9.3	9.1	8.4	8.1	8.4	8.2	7.9	8.2	7.9	7.5	8.4
3.5	9.7	9.2	8.4	------	------	9.6	9.5	8.7	8.7	8.7	8.5	8.3	8.5	8.6	8.7	9.0	9.0	9.0	9.1	9.2	8.5	8.5	8.6	8.4	8.0	8.2	7.9	7.5	8.3
4.0	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	8.8	9.0	8.9	------	8.6	8.6	8.7	8.6	8.2	8.0	8.0	7.6	8.2
4.5	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	8.9	8.8	------	8.7	8.8	8.7	8.7	8.3	7.9	8.1	7.7	8.3
5.0	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	8.9	8.8	------	8.8	8.9	8.8	8.7	8.5	7.7	8.2	7.7	8.4

Depth	1920 Jan. 10	Feb. 7	Mar. 13	Apr. 24	May 8	May 29	June 18	July 17	Aug. 16	Sept. 17	Oct. 25	1921 Jan. 15	Feb. 22
Bot.	2.5	3.1	3.3	4.8	5.9	8.9	10.2	10.5	10.8	11.0	12.3	3.1	3.2
0.5	3.6	4.0	4.4	4.8	5.8	7.4	9.1	9.7	10.0	10.0	11.1	4.8	4.4
1.0	5.0	5.4	5.3	5.2	5.9	6.6	7.9	8.7	9.9	9.3	9.7	6.0	5.5
1.5	6.5	6.5	6.3	5.9	6.1	6.6	7.3	8.0	8.3	8.6	8.9	7.0	6.4
2.0	7.4	7.3	6.9	6.8	6.7	6.8	7.4	7.5	7.7	8.2	8.3	7.8	7.2
2.5	7.9	7.8	7.4	7.4	7.2	7.1	7.4	7.4	7.5	7.8	7.9	8.1	7.7
3.0	8.0	8.2	7.8	7.8	7.7	7.5	7.6	7.5	7.4	7.6	7.8	8.3	7.9
3.5	8.1	8.4	8.0	8.0	8.0	7.8	8.2	7.6	7.5	7.7	7.7	8.2	8.0
4.0	8.1	8.5	8.1	8.3	8.2	8.0	8.3	7.7	7.6	7.8	7.6	8.2	8.0
4.5	8.1	8.6	8.3	8.4	8.4	8.2	8.4	7.9	7.8	7.9	7.6	8.2	8.0
5.0	8.2	8.6	8.4	8.5	8.4	8.3	8.5	8.0	8.0	8.0	7.8	8.2	8.0

TABLE 20. *Observations at Station II, 23.5 meters*

Depth	1917 Jan. 5	Feb. 16	Mar. 10	Sept. 15	Dec. 22	1918 Feb. 9	Mar. 2	Mar. 23	May 23	June 25	July 18	Aug. 7	Sept. 7	Oct. 23	Nov. 27	1919 Jan. 7	Feb. 8	Mar. 11	Apr. 22	May 8	May 27	June 13	July 12	Sept. 4	Sept. 23	Oct. 11	Oct. 31	Nov. 19
Bot.	3.0	3.0	3.9	13.8	2.3	3.1	3.6	3.8	10.6	11.4	12.0	12.5	13.1	13.4	6.6	1.5	3.1	3.9	5.9	8.1	9.0	9.9	9.8	9.8	10.0	10.6	10.7	5.6
0.5	4.4	4.4	4.3	12.2	4.5	4.0	4.0	4.0	8.0	9.7	11.2	11.5	11.5	12.2	9.4	5.7	4.9	5.8	5.7	7.2	8.4	9.1	9.1	9.3	9.7	9.9	10.1	7.5
1.0	6.7	5.8	5.2	11.2	6.8	5.2	5.0	4.4	7.4	8.7	9.7	10.3	10.2	10.9	10.4	7.7	6.4	6.6	5.7	6.5	7.4	8.5	8.6	8.7	9.1	9.3	9.5	8.5
1.5	8.3	6.8	6.4	10.0	8.2	6.6	6.0	5.3	7.5	8.2	8.8	9.2	9.2	10.2	10.0	8.4	7.7	7.7	6.3	6.6	7.1	7.8	8.1	8.3	8.6	8.7	9.0	8.8
2.0	9.0	7.8	7.4	9.2	8.9	7.4	6.9	6.0	7.9	8.1	8.4	8.5	8.7	9.2	9.5	8.9	8.6	8.4	7.1	7.1	7.1	7.3	7.9	8.0	8.3	8.3	8.6	8.3
2.5	9.4	8.4	8.0	8.6	9.3	8.4	7.4	6.6	8.3	8.5	8.4	8.1	8.3	8.6	9.0	8.8	8.8	8.7	7.7	7.7	7.5	7.5	7.8	7.9	8.1	8.0	8.3	8.3
3.0	9.4	8.8	8.3	8.2	9.4	9.1	7.7	7.3	8.6	8.6	8.4	7.9	8.2	8.4	8.6	8.6	8.9	8.8	8.1	8.2	7.8	7.7	7.9	7.8	8.0	7.8	8.2	8.3
3.5	9.4	9.0	8.6	7.9	9.4	9.3	8.0	7.5	8.8	8.7	8.3	7.8	8.2	8.3	8.4	8.5	8.8	8.9	8.5	8.3	8.1	8.1	8.1	8.0	7.9	7.6	8.0	8.3
4.0	------	------	------	------	------	------	------	------	------	------	------	------	------	------	8.3	8.4	8.7	------	8.6	8.4	8.3	8.4	8.2	8.1	8.0	7.5	8.0	8.2
4.5	------	------	------	------	------	------	------	------	------	------	------	------	------	------	8.2	8.3	8.7	------	8.7	8.6	8.4	8.5	8.3	5.2	8.0	7.5	8.1	8.2
5.0	------	------	------	------	------	------	------	------	------	------	------	------	------	------	8.1	8.3	8.5	------	8.8	8.6	8.5	8.5	8.4	------	8.1	7.4	8.1	8.2

Depth	1920 Jan. 10	Mar. 13	Apr. 20	May 8	May 29	June 18	July 17	Aug. 14	Sept. 17	Oct. 25	1921 Jan. 15	Feb. 22
Bot.	2.9	3.0	3.9	5.8	9.4	9.9	10.0	10.2	10.5	10.9	2.7	3.2
0.5	4.0	4.2	4.2	5.5	7.9	8.9	9.4	9.8	9.7	10.1	4.0	3.7
1.0	5.7	5.0	5.0	5.6	6.5	7.6	8.3	8.7	8.9	9.3	5.6	4.5
1.5	6.8	6.0	5.9	6.0	6.3	6.9	7.5	7.8	8.1	8.6	6.7	5.5
2.0	7.5	6.6	6.7	6.6	6.7	6.9	7.1	7.3	7.7	8.0	7.4	6.5
2.5	7.9	7.2	7.2	7.1	7.0	7.0	7.0	7.1	7.5	7.7	7.8	7.0
3.0	7.9	7.5	7.7	7.4	7.3	7.2	7.1	7.2	7.4	7.6	7.9	7.4
3.5	8.0	7.7	7.9	7.6	7.6	7.6	7.3	7.3	7.4	7.5	8.0	7.6
4.0	8.0	7.9	8.2	8.0	7.7	2.8	7.6	7.5	7.5	7.6	7.8	7.7
4.5	8.0	8.0	8.3	8.1	8.0	8.0	7.8	7.6	7.6	7.6	7.7	7.8
5.0	7.9	8.0	8.4	8.2	8.1	8.1	8.0	8.0	7.7	7.7	7.7	7.8