ON THE EMPLOYMENT OF THE METHOD OF LEAST SQUARES IN THE REDUCTION OF TRANSIT OBSERVATIONS.

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In longitude work upon the continent of Europe the polar stars employed to find the azimuth and collination of the transit are usually within a few degrees of the north pole. Each such star gives a value of the azimuth, and three are generally observed each evening, in reversed positions of the instrument; while the time-stars are, according to the instructions, twenty or more in number, arranged symmetrically about the polars, in such manner as to give opportunity for two exchanges of signals. The declinations of these time-stars are arranged at the same zenith distances south, as the polars are north of the zenith, so as to eliminate pivot-errors.

This arrangement, for latitudes near 50°, combines the following advantages:

1. The polar stars are slow enough in their apparent motions for a deliberate reversal during each of their transits.

2. The time-stars are in the declinations most favorable to accurate observation when they are equally distant from the zenith with the polars.

3. The reduction becomes extremely simple in execution, as the different instrumental corrections are directly determined and equations of conditions, to be solved by the method of least squares are not required.

For the more southern latitudes of Europe, which correspond to those of the northern part of the United States, these three conditions cannot be simultaneously fulfilled. If, for instance, the second condition be insisted upon, the first falls out, for latitude 43°, and *vice versa*. In order to fulfill this second condition the average declination of time stars must be taken at about 20°; and the polar-stars at 66°: which is too far south to accomplish the necessary reversals without hurry during the single transits.

Certain limitations, however, which are necessary in Europe, are needless in America. For instance, the greater length of the circuits over which very much of our telegraphic longitude work is done, renders it very difficult to observe the same stars at both stations. As our absolutely definitive reductions are not immediately necessary, the star-places used in any campaign can be left open for future improvement; when, for instance, geodetic measures shall be made over the whole territory. Again, the unsettled country in which much of the work is done renders it necessary for us to

adopt a different type of instruments from the European, lighter and more portable, less liable to injury in transportation; but, on the other hand, more care has to be exercised in so arranging our star-lists that all conceivable instrumental errors shall be eliminated; we cannot depend too f[illegible] upon one particular arc of the circle described by the telescope. A[illegible]nese things considered, it has been the practice of American observers to select their time and polar stars in a much wider range of declination than the Europeans employ; and to use the method of least squares in their reductions.

I have myself employed these methods, both of selection of stars and reduction, in second longitude campaigns; and have formulated for myself the practical rules to be followed. The earlier years of this work were spent in determinations at very distant points, accessible with difficulty, and with quite inferior instruments; so that, as they were also usually in bad repair, it required very careful handling to produce even tolerable results.

Later, in 1878, I had the use of an excellent transit instrument, which was in first-rate condition; and was then able to apply the rules which I had previously made for myself, to a better state of external circumstances. In the present paper I shall state the conditions for a good series of time-determinations in this country, as I now understand them, with especial reference to the western States and Territories, and the methods of reduction.

The observations to which I have last referred were made in 1878, at Ogden, Utah, in latitude 41° 13′. The results are given in the Report of the Chief of Engineers for 1879–80 (Executive Documents, 2d Session, 46th Congress, Vol. 5), pages 1983–5. It will be seen that during a space of 2½ months, from July 25th to October 10th, 74 days in all, there were 67 determinations of time. Signals were exchanged on 26 nights with the regular observers of Captain Wheeler's survey; and the observations on these nights reduced by the method of least squares; all these calculations, as well as the preliminary ones necessary for the other nights, were completed within the two months after the first exchange, August 12th, so that the method of reduction is not too complicated to be practicable.

The following would be, in my judgment, an advantageous selections of stars for a latitude 41° — 44°; the group is arranged nearly as at Ogden, and differs from my ordinary arrangements in those times only because I had then fewer close polar stars available. The stars are taken from the Berlin Jahrbuch, or other ephemerides, and from my catalogue of 2018 stars published by the Engineer Bureau:

Name of Star.	A. R. 1885.0	Decl. 1885.0	Remarks.
Cassiopeiæ	0 26 28	+ 62 18	
π Andromedæ.....	0 30 44	+ 33 5	
55 Piscium	0 33 52	+ 20 48	
o Cassiopeiæ	0 38 19	+ 47 39	
δ Piscium	0 22 43	+ 6 57	
Cephei 43 H.....	0 53 12	+ 85 38	Reverse on this star.
ε Piscium	1 2 27	+ 5 2	
τ Piscium	1 5 20	+ 29 29	
α Piscium	1 7 30	+ 23 59	
f Piscium.....	1 11 52	+ 3 1	
ψ Cassiopeiæ	1 17 41	+ 67 31	

It will be noticed that I have introduced no stars at lower culmination. The reason for this is, that unless they are very near the pole, the azimuth co-efficient which they furnish is positive in sign, and the stars which balance them as to the sign of this co-efficient do not as to the form of the pivots. In other words, a combination of stars below and above pole would give the clock correction with a large probable error and with a danger of constant error, while a combination of stars below pole with those at corresponding south zenith distances would be far less advantageous, owing to the identity of sign in the azimuth co-efficient, and its greater magnitude for the southern stars, than a combination of stars above pole on the north side with the ordinary northern time-stars.

When the least square reduction is made, the theoretical test of a good combination is obtained by Jacobi's theorem, which I quote below; the practical test by the quotient.

$$\frac{\text{Weight of clock-correction.}}{\text{Number of transits}}$$

which should approach the weight of one transit of a zenith star: Practically I usually employ the following formula for the weight or of one complete transit (on 5 or more wires, or 7 or more, according to the way the diaphragm is arranged).

$$w = \frac{1.3}{1 + 0.3 \text{ Sec.}^2\delta}$$

For latitude 42° 42′ 5, this gives the weight of a zenith star equal to 0.836; in general it is easy so to arrange the groups that the weight of the clock correction from 10 stars shall be 7 at least; but where lower culminations and southern stars are freely employed this number frequently is reduced to 5 or less. That is to say, 10 stars well arranged give as good a clock correction as 14 ill arranged; and waste of time spent in computation is thus avoided. The few extra minutes lost in observations are usually of less account; and are perhaps compensated by the greater pains necessary beforehand. In order to modify the weight-formula to advantage more

observations are needed, which I intend to make with a portable instrument now constructing for Williams College Observatory; the field observations have confirmed it with sufficient certainty for my purposes. For in my least square reductions I have pursued the course so strongly recommended by Gauss (Astr. Nachrichten v. 5, cols. 227 following; Méthode des moindres carrés, mémoires traduits par J. Bertrand, p. 163) of determining in advance approximate values of the unknowns, and using the least squares only to find minute corrections. And the following are the results for the clock-corrections of the dates in the Ogden campaign, as published in the Report before alluded to:

Date.	Correction to provisional result.	Date.	Correction to provisional result.
	s.		s.
Aug. 12.....	−0.008	Sept. 20.....	+0.001
19.....	−0.021	21.....	−0.020
21.....	−0.003	22.....	+0.001
26.....	−0.010	26.....	+0.001
28.....	−0.006	27.....	−0.008
Sept. 5.....	+0.016	30.....	+0.011
7.....	+0.010	Oct. 1.....	−0.002
8.....	0.000	3.....	−0.009
10.....	−0.010	5.....	−0.005
11.....	+0.017	6.....	−0.005
12.....	+0.009	7.....	−0.003
13.....	+0.010	8.....	−0.013
17.....	−0.031	10.....	−0.007

The total effect upon the longitudes of the application of least squares, consists in the differences of their corrections for the stations. The observors at the corresponding stations in this campaign had worse instruments than I, and were not so scrupulous in arranging their star lists; so that the corresponding corrections came out rather larger. Their time determinations did not suffer sensibly from this cause, however; as both were experienced observers. The average without regard to sign of the 26 numbers given above — each of which, with trifling exceptions is the result of two groups of stars — is $0.^s 0091$: and the probable error of one preliminary solution $\pm 0.^s 0077$; or for a single group of 10 stars $\pm 0.^s 011$.

It is evident, then, that if the weights are assigned by my formula with approximate corrections — so, for instance as to make sufficient distinction between a very slow-moving and a quiet-moving star — there need be no fear that it is not correct enough. The objections to the old process of giving all stars a weight unity is that the polars of high declination dominate too much in azimuth; this process is about equivalent to leaving out the quicker-moving stars north of 65° altogether.

Another practice which is quite common is altogether meaningless. When the weights are approximately correct there is no necessity of omit-

ting the polar stars in arranging the final table of clock corrections. Each star, quick-or slow-moving, should go in, with its proper weight to the clock-correction The method of least squares gives, namely: the *most probable* values of all the unknowns; and a *more probable* one than this cannot be obtained in any other way, unless the original system of weights be altered. But this would involve a contradiction; the weight, according to definition, refers not to the use to be made of the observation, but to the measure of its precision. The following example taken from my work on the longitude campaign of 1878 will illustrate several points of this paper.

The observer at Fort Bliss (latitude 31° 45′) frequently inserted stars subpole in his determinations. As a consequence the weight of the final time determination was less, proportionately, than it otherwise would have been; for a zenith star my formula would give

$$\frac{1.3}{1+0.\,z\sec^2 31°\,45'} = \frac{1.3}{1.415} = 0.919$$

But the group after signals on September 8th contains 11 stars, giving a final weight of 6.92; or about that of 8 or 9 stars nearer the zenith; two stars are below pole, and therefore less advantageous. The results of the individual stars (those north of 65° are marked with an asterisk; these are now between 45° and 65°) are as follows:

	Clock Corr.	Clamp.
	m s	
θ Aquarii	−1 24.79	W
γ Aquarii	−1 24.89	W
π Aquarii	−1 24.82	W
Draconis 9 H. sp	−1 25.28*	W
Groomb. 3834	−1 25.39*	W
ζ Pegasi	−1 24.75	W
λ Pegasi	−1 24.83	E
τ Cephei	−1 25.07*	E
Bradley 1508 sp	−1 25.37*	E
o Andromedae	−1 25.09	E
α Pegasi	−1 24.74	E

The least square solution gives the mean clock-correction −1′ 24″.888, but this includes, with their proper weight, the stars marked with an asterisk. If we omit these, and give all others a weight unity we shall have −1′ 24″.844, or +0″.044 more. But it is plain from least square solution that some cause affects the observations north of the zenith, probably wear of pivots [1]; and the mean of the 6 stars south of the zenith gives −1′ 24″.803;

[1] This is confirmed by the other dates when the same instrument was employed.

the three north of the zenith and above pole $-1'\ 25''.183$; and the zenith distances are nearly equal for these two groups. It is plain from this that the least square value $-1'\ 24''.888$ is probably nearer correct than the greater (algebraic) value $-1'\ 24''.803$; as it ought to be. In other words, the exclusion from the final clock corrections of the quick moving polar stars renders the elimination of pivot error very questionable; as, in fact, does the employment of groups not sufficiently well-balanced.

The simpler method of reduction, which gives the instrumental corrections separately, without least squares, really assigns to the polar stars their proper weight; especially if they are all observed at upper culmination, as the German instructions recommend. For in this case the final clock corrections from each time star are actually reduced by interpolation to the zenith, as may be seen by using Dr. Braun's graphical method given in vol. 109 (No. 2595) of the Astronomische Nachrichten. So that in the previous example it is not surprising that the preliminary solution gave a value ($1'\ 24''.90$) very nearly equal to the final result. Nor is it very plain that the least square method is absolutely indispensable; I have employed it in my reductions without objection, although I have never known a case in which it materially aided in producing a better agreement in the resulting longitude. The chief argument in its favor here is that it renders it unnecessary to restrict the selection of stars to a narrow range, while it removes the arbitrary character of the reductions when this restriction has not been carried into effect. The chief objection to the method of least squares is that observers who are not both experienced and careful sometimes permit a blind faith in it to mislead them in preparing their working lists; and forget the necessity of making observations enough and of the right quality, which is no less when least squares are used in reductions than at any other time.

Jacobi's theorem is in substance the following; and bears directly upon the point in question:

In order to obtain the least square result for any unknown of m equations with a less number of unknowns, we solve all possible combinations n by n of the m equations, and multiply each such result for this unknown by the square of the corresponding determinant. We add all these products together and divide their sum by the sum of the squares of the determinant factors; the quotient will be the least square result sought for.[1]

To a least square result, then, unfavorable combinations furnish small contributions relatively to favorable ones; and if the favorable ones cannot be made, the unfavorable ones are better than nothing. Stars below pole are the proper ones to employ in combination with those above, when the azimuth is the unknown most needed, as in setting up a meridian mark; but for time-determinations proper the best combination is that of polars above pole with time-stars at nearly equal zenith distances on the other side.

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[1] Jacobi, De Formatione et Proprietatibus Determinantium: Crelle's Journal.

I have annexed to this paper, a table for Ogden observatory, latitude 41° 13′ 8″.6, which is calculated according to the formula.

$$p = \frac{1.3}{1 + 0.3 \sec^2 \delta}$$

$$A = \frac{\sin(\varphi - \delta)}{\cos \delta}$$

$$C = \frac{1}{\cos \delta}$$

The columns p, C p, C_2 p hold good for any station; the values of A p, A_2 p, A C p must be recalculated for every new latitude. But I have not omitted them, as their values will serve as a control for a good many places near the same parallel. The numbers are given to four decimals, to enable the computor to interpolate readily to three.

The 52 nights out of 78 upon which signals were not exchanged at Ogden, afforded an excellcnt opportunity for the criticism of the instrument. A peculiarity of much far western longitude work is that the telegraph lines are in bad order much of the time, and do not afford opportunities for long exchanges. On some nights of the exchanges with Fort Bliss (Texas), it was impossible to get a circuit through on the more direct route, by way of Cheyenne, Denver, and Santa Fé, and recourse was had to a line of 2,000 miles in length via San Francisco, Southern California, and Arizona. The time expended while waiting is, under such circumstances, best applied to the study of the peculiarities of the instrument.

In conclusion, I will indicate the principal points of this paper as a summary of directions for the determination of time in latitudes 25° to 49°.

1. A group of time-stars should contain about 10 stars in all, with one reversal.

2. It should contain no more than two or three polars; one of which may be within 10 degrees of the pole.

3. Stars north of the zenith, not polars, should be included; and may take the place of polars to some extent.

4. Lower culminations should in general be avoided; and time-stars far south of the zenith.

5. The stars of a group should so balance at the zenith that the co-efficient of azimuth and collination in the normal equation for clock-correction should be very small, and the weight of the clock-correction large in proportion to the number of stars.

6. With proper blank forms the preliminary reduction without least squares can be very simply effected, and this should always be done.

7. The least square deduction is of little benefit except in removing the effect of any arbitrary assumptions in the previous process; and should be

employed only for the signal-nights, or other times when the last degree of precision is required.

8. In making the least square reductions the weights at different declinations should be combined with the coefficients for azimuth and collination; and the unknowns should be in the form of corrections to the values derived from the preliminary reduction.

JACOBI'S THEOREM.

NOTE.—[I give this in the original Latin, as it is very important, and seems not to be well known to mathematicians.]

Proponantur aequationes:

$$ax+a'x,+a''x_2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots +a^{(n)}x_n = l$$

$$a_1x+a'_1x,+a''_1x_2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots +a,^{(n)}x_n = l_1$$

$$a_p x+a'_p x_1+a''_p x_2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots +a_p{}^{(n)}x_n = l_p$$

Quarum numerus incognitarum numerum excedat; e quolibet systemate n+1 aequationum praecedentium valor incognitae eruatur atque per quadratum Determinantis eius systematis, RR, multiplicetur; quibus factis pro singulis aequationum propositarum combinationis omnium illorum productorum summa per summam omnium RR dividatur: eruitur incognitae valor idem atque invenitur, si aequationes propositae per Methodum Minimorum Quadratorum tractantur."

Observandum est, valores omnium incognitorum qui ex eadem aequationum propositarum combinatione proveniant secundum Prop. praec. per *eandem* quantitatem RR multiplicare, quam ideo in applicationibus ad *Methodum Minimorum Quadratorum convenit appellare Pondus Combinationis*, a pondore valoris incognitae bene distinguendum.

The theorem is thus given in Crelle, vol. 22, p. 316; it is also given independently by Mr. J. W. L. Glaisher in Vol. 40, of the Monthly Notices, p. 607.

SQUARES AND PRODUCTS OF LEAST SQUARE CO-EFFICIENTS FOR TIME — OGDEN.

Decl.	P		A p		C p		A_2 p		A C p		C_2 p	
0°	1.0000		+.6589		+1.0000		.4342		+.6589		1.0000	
		— 1		131		+ 1		—172		—130		+ 2
1	.9999		.6458		1.0001		.4170		.6459		1.0002	
		2		133		2		168		130		7
2	.9997		.6325		1.0003		.4002		.6329		1.0009	
		3		134		4		166		129		12
3	.9994		.6191		1.0007		.3836		.6200		1.0021	
		5		134		6		164		129		16
4	.9989		.6057		1.0013		.3672		.6071		1.0037	
		7		136		8		160		127		22
5	.9982		.5921		1.0021		.3512		.5944		1.0059	
		7		137		9		158		128		26
6	.9975		.5784		1.0030		.3354		.5816		1.0085	
		10		138		10		155		127		31
7	.9965		.5646		1.0040		.3199		.5689		1.0116	
		10		139		12		152		128		35
8	.9955		.5507		1.0052		.3047		.5561		1.0151	
		11		140		14		148		127		41
9	.9942		.5367		1.0066		.2897		.5434		1.0192	
		13		141		16		147		128		48
10	.9929		.5226		1.0082		.2750		.5306		1.0240	
		15		143		17		144		128		48
11	.9914		.5083		1.0099		.2606		.5178		1.0288	
		17		144		19		141		129		56
12	.9897		.4939		1.0118		.2465		.5049		1.0344	
		19		145		20		139		129		61
13	.9878		.4794		1.0138		.2326		.4920		1.0405	
		19		147		22		135		130		67
14	.9859		.4647		1.0160		.2191		.4790		1.0472	
		22		148		24		133		132		71
15	.9837		.4499		1.0184		.2058		.4658		1.0543	
		23		149		25		130		133		78
16	.9814		.4350		1.0209		.1928		.4525		1.0621	
		25		151		27		127		134		83
17	.9789		.4199		1.0236		.1801		.4391		1.0704	
		27		153		29		123		136		89
18	.9762		.4046		1.0265		.1678		.4255		1.0793	
		28		153		30		121		138		95
19	.9734		.3893		1.0295		.1557		.4117		1.0888	
		31		156		31		117		140		101
20	.9703		.3737		1.0326		.1440		.3977		1.0989	
		32		157		33		115		142		107
21	.9671		.3580		1.0359		.1325		.3835		1.1096	
		34		159		35		110		145		114
22	.9637		.3421		1.0394		.1215		.3690		1.1210	
		36		160		36		107		147		121
23	.9601		.3261		1.0430		.1108		.3543		1.1331	
		38		162		38		104		151		127
24	.9563		.3099		1.0468		.1004		.3392		1.1458	
		41		164		39		100		154		135
25	.9522		.2935		1.0507		.0904		.3238		1.1593	
		42		166		40		95		157		142
26	.9480		.2769		1.0547		.0809		.3081		1.1735	
		45		168		42		92		162		149
27	.9435		.2601		1.0589		.0717		.2919		1.1884	
		47		170		43		87		165		158
28	.9388		.2431		1.0632		.0630		.2754		1.2042	
		50		171		45		83		170		165
29	.9338		.2260		1.0677		.0547		.2584		1.2207	
		52		174		45		79		175		174

SQUARES AND PRODUCTS OF LEAST SQUARE CO-EFFICIENTS FOR TIME — OGDEN.

Decl.	P	A p	C p	A_2 p	A C p	C_2 p
30	.9286	.2086	1.0722	.0468	.2409	1.2381
	55	175	47	72	180	183
31	.9231	.1911	1.0769	.0396	.2229	1.2564
	58	178	48	68	185	191
32	.9173	.1733	1.0817	.0328	.2044	1.2755
	60	180	49	63	192	201
33	.9113	.1553	1.0866	.0265	.1852	1.2956
	63	182	50	57	197	211
34	.9050	.1371	1.0916	.0208	.1655	1.3167
	66	183	51	51	205	221
35	.8984	.1188	1.0967	.0157	.1450	1.3388
	70	186	52	43	211	232
36	.8914	.1002	1.1019	.0114	.1239	1.3620
	73	188	52	39	219	242
37	.8841	.0914	1.1071	.0075	.1020	1.3862
	76	189	52	31	227	254
38	.8765	.0625	1.1123	.0044	.0793	1.4116
	79	192	53	23	236	265
39	.8686	.0433	1.1176	.0021	.0557	1.4381
	84	194	53	17	245	+278
40	.8602	.0239	1.1229	+.0007	.0312	1.4659
40°	.8602	+.0239	+1.1229	.0006	+.0312	1.4659
	—87	—196	+54	— 6	— 225	+ 291
41	.8515	+.0043	1.1283	.0000	+.0057	1.4950
	91	197	53	+ 3	265	304
42	.8424	—.0154	1.1336	.3003	—.0208	1.5254
	95	200	52	12	276	317
43	.8329	.0354	1.1388	.0015	.0484	1.5571
	100	201	52	22	288	332
44	.8229	.0555	1.1440	.0037	.0772	1.5903
	104	203	50	34	300	347
45	.8125	.0758	1.1490	.0071	.1072	1.6250
	109	204	50	44	312	362
46	.8016	.0962	1.1540	.0115	.1384	1.6612
	113	205	48	57	327	379
47	.7903	.1167	1.1588	.0172	.1711	1.6991
	119	207	45	71	342	395
48	.7784	.1374	1.1633	.0243	.2053	1.7386
	123	207	44	83	357	412
49	.7661	.1581	1.1677	.0326	.2410	1.7798
	129	208	40	99	373	430
50	.7532	.1789	1.1717	.0425	.2783	1.8228
	135	208	37	114	390	449
51	.7397	.1997	1.1754	.0539	.3173	1.8677
	140	208	33	131	408	468
52	.7257	.2205	1.1787	.0670	.3581	1.9145
	147	207	28	148	427	487
53	.7110	.2412	1.1815	.0818	.4008	1.9632
	152	207	23	168	447	508
54	.6958	.2619	1.1838	.0986	.4455	2.0140
	158	205	17	187	469	529
55	.6800	.2824	1.1855	.1173	.4924	2.0669
	165	203	10	208	489	549

SQUARES AND PRODUCTS OF LEAST SQUARE CO-EFFICIENTS FOR TIME — OGDEN.

Decl.	P		Ap		Cp		A_2p		ACp		C_2p	
56	.6635	172	.3027	200	1.1865	+ 2	.1381	231	.5413	513	2.1218	571
57	.6463	178	.3227	197	1.1867	−6	.1612	254	.5926	536	2.1789	594
58	.6285	184	.3424	193	1.1861	16	.1866	279	.6462	561	2.2383	615
59	.6101	192	.3617	188	1.1845	27	.2145	305	.7023	587	2.2998	638
60	.5909	198	.3805	182	1.1818	38	.2450	333	.7610	613	2.3636	661
61	.5711	205	.3987	174	1.1780	52	2783	362	.8223	640	2.4297	684
62	.5506	212	.4161	166	1.1728	67	.3145	392	.8863	668	2.4981	705
63	.5294	218	.4327	157	1.1661	83	.3537	423	.9531	697	2.5686	728
64	.5076	225	.4484	145	1.1579	100	.3960	457	1.0228	725	2.6414	748
65	.4851	230	.4629	133	1.1479	119	.4417	490	1.0953	754	2.7162	769
66	.4621	236	.4762	119	1.1360	139	.4907	526	1.1707	784	2.7931	788
67	.4385	242	.4881	102	1.1221	161	.5433	561	1.2491	812	2.8719	804
68	.4143	246	.4983	85	1.1060	186	.5994	598	1.3303	840	2.9523	820
69	.3897	250	.5068	66	1.0874	211	.6592	635	1.4143	867	3.0343	834
70	.3647	253	.5134	43	1.0663	238	.7227	672	1.5010	894	3.1177	843
71	.3394	255	.5177	−21	1.0425	268	.7899	710	1.5904	918	3.2020	850
72	.3139	256	.5198	+ 5	1.0157	297	.8609	746	1.6822	940	3.2870	854
73	.2883	256	.5193	33	.9860	329	.9355	781	1.7762	959	3.3724	853
74	.2627	254	.5160	62	.9531	363	1.0136	816	1.8721	975	3.4577	846
75	.2373	251	.5098	94	.9168	396	1.0952	847	1.9696	988	3.5423	837
76	.2122	246	.5004	127	.8772	431	1.1799	877	2.0684	996	3.6260	819
77	.1876	239	.4877	162	.8341	466	1.2676	903	2.1680	998	3.7079	797
78	.1637	232	.4715	198	.7875	501	1.3579	925	2.2678	997	3.7876	767
79	.1407	220	.4517	− 234	.7374	− 536	1.4504	943	2.3675	988	3.8643	782
80	.1187		−.4283		.6838		1.5447		−2.4663		3.9375	

SQUARES AND PRODUCTS OF LEAST SQUARE COEFFICIENTS FOR TIME — OGDEN.

Decl.	P	A p	C p	$A_2 p$	A C p	$C_2 p$
80°	0.1187	—.4283	+.6838	1.5447	—2.4663	3.9376
	—207	+ 273	—570	+ 956	+ 973	+ 689
81	0.0980	.4010	.6268	1.6403	2.5936	4.0065
	192	310	603	963	951	640
82	0.0788	.3700	.5665	1.7366	2.6587	4.0705
	175	347	633	964	923	584
83	0.0613	.3353	.5032	1.8330	2.7510	4.1289
	163	385	662	958	888	521
84	0.0457	.2968	.4370	1.9288	2.8398	4.1810
	12	419	686	945	844	454
85	0.0321	.2549	.3684	2.0233	2.9242	4.2264
	114	454	709	925	795	378
86	0.0207	.2095	.2975	2.1158	3.0037	4.2642
	89	484	728	897	738	299
87	0.0118	.1611	.2247	2.2055	3.0775	4.2941
	65	513	741	865	676	217
88	0.0053	.1098	.1506	2.2920	3.1451	4.3158
	40	538	751	826	608	131
89	0.0013	.0560	+0.0755	2.3742	3.2059	4.3289
	—13	560	755	776	536	+44
90	0.0000	0.0000	0.0000	2.4518	3.2595	4.3158
	+13	577	755	722	460	—44
89 sp	0.0013	+0.0577	—0.0755	2.5240	3.3055	4.3289
	40	590	751	664	381	131
88 sp	0.0053	.1167	.1506	2.5904	3.3436	4.3158
	65	599	741	603	301	217
87 sp	0.0118	.1766	.2247	2.6507	3.3737	4.2941
	89	603	728	634	220	299
86 sp	0.0207	.2369	.2975	2.7041	3.3957	4.2642
	114	603	709	467	140	378
85 sp	0.0321	.2972	.3684	2.7508	3.4097	4.2264
	136	598	686	397	+60	554
84 sp	0.0457	.3570	.4370	2.7905	3.4157	4.1810
	156	591	662	327	—15	521
83 sp	0.0613	.4161	.5032	2.8232	3.4142	4.1289
	175	578	633	257	88	584
82 sp	0.0788	.4739	.5665	2.8489	3.4054	4.0705
	192	564	603	187	158	640
81 sp	0.0980	.5303	.6268	2.8676	3.3896	4.0065
	+ 207	+ 544	—570	+ 121	—222	—689
80 sp	0.1187	+.5847	—.6838	2.8797	—3.3674	3.9376