POINT-SAMPLING METHOD OF MEASURING BASAL AREA

A more rapid but yet accurate method of sampling tree basal area and also for obtaining tree volumes was devised by Bitterlich (1948). This method does not require measurement of sample plot areas nor does it require measurement of tree diameters. It is called the Bitterlich system, but is also commonly referred to as point-sampling, variable plot sampling, plotless cruising or plotless timber estimating. Reference to this method will be called point-sampling in this publication.

FIGURE 1. Point-sampling with angle-gauge.
The basic instruments used in the point-sampling method are the angle-gauge and wedge prism. A cruiser counts the number of trees around a sampling point whose diameters at breast height appear larger than the cross-arm of the angle-gauge (Fig. 1). Using a wedge prism, he counts the number of trees whose stem sections at breast height when viewed through the prism do not appear to be detached from the main stem (Fig. 2). If the tree count is multiplied by a predetermined factor (basal area factor—BAF) the basal area per acre is obtained at that particular sampling point. There is a definite correlation between tree basal area and tree volumes, therefore timber estimates may be obtained when counted trees are tallied by merchantable height.
classes or when the total number of logs or pulpwood bolts in the counted trees are recorded. The number of points to be taken to obtain reliable basal area and volume estimates is discussed later.

The predetermined basal area factor can be any convenient factor based upon the size angle that one selects to use for his gauge or by the strength of the wedge prism. Timber size will determine which factor is most useful. In the average sawtimber stand in the Lake States, an instrument with a basal area factor of 10 is recommended. In uniform or dense pole timber stands the 10 factor is also recommended. On the other hand, if one is working with large sawtimber, a 20-factor or larger might be the most convenient. In light density pole stands a 5-factor instrument may prove to be the most satisfactory since it will reach out farther for the small trees. The use of an instrument having an unwieldy basal area factor can lead to a waste of time, confusion in ascertaining “in” and “out” trees, and unreliable estimates.

The main criteria in selecting a specific basal area factor are stand density and the size of the trees normally involved. In dense pole timber stands some trees are obscured by others, and in large timber the larger trees may be a great distance from the point center, resulting in some trees being overlooked and not counted. For example, using an instrument with a basal area factor of 10, a 36-inch tree is counted up to a distance of 99 feet from the center. One can readily see that such trees can be easily missed in areas where there is any amount of underbrush.

**Instruments Used**

Basically there are two types of instruments used in point-sampling. One is the *horizontal angle-gauge* and the other the *wedge prism*. Angle-gauges are nothing new to the forester; for example, the hypsometer for measuring tree heights is a vertical angle-gauge, and the Biltmore stick used in measuring tree diameters is a horizontal angle-gauge. Both of these instruments are based upon geometric principles centuries old.

a. A simple *stick-type angle-gauge* with a basal area factor of 10, for instance, can be constructed by mounting a one-inch crossarm at the end of a 33-inch stick. A peephole sight may be mounted at the other end. The angle created by the eye (as the vertex) at the peephole and the outer edges of the crossarm is 104.18 minutes. The ratio of the width of crossarm to the length of stick is 1:33. Likewise the ratio of the tree diameter of a tree just covered by the crossarm and the distance from tree to observer is 1:33 (an example of similar triangles.) In other words, the crossarm will exactly cover a 12-inch (1-foot) tree at 33 feet, a 24-inch (2-foot) tree at 66 feet, etc. (Fig. 3). This ratio of 1:33 gives the instrument a basal area factor of 10 meaning that each counted tree then represents 10 square feet of basal area.

The crossarm and stick of an angle-gauge having a basal area factor of 10 can be of any size as long as this ratio of 1:33 is maintained. The width of
the crossarm per inch of stick is obtained by dividing 1 by 33, or .0303 inches. For a 20-inch stick, for example, the crossarm would have to be .0303 inches × 20 or .606 inches. A convenient, all-around gauge can be made by mounting hinges on a folding hypsometer (Fig. 4). A gauge can be constructed of any length and for any predetermined basal area factor. However, gauges less than 20 inches in length are not recommended because of the difficulty of focusing on a far object and a near object at the same time. The image of the crossarm against the tree becomes blurred as the stick is reduced in length below 16–20 inches. Adjustable or combination angle-gauges can be constructed for two or more factors (Tables 2 and 3). Allowances must be made for the distance from the eye to the end of the stick in figuring the effective length of the gauge (Fig. 4).

b. The Spiegel-Relaskop is another of the horizontal-type angle-gauges and also measures tree heights, diameters, distance and slope. It has four built-in angle-gauges and is the most accurate instrument that can be used for determining basal area since it automatically corrects for slope. The cost of this instrument will probably limit its being brought into common usage for the present.
The Spiegel Relaskop and a demonstration of its use in the field.
Construction of Bitterlick Stick

Computation for Gauge Widths

| Total length of Bitterlick Stick | 25.468" |
| Eye relief displacement           | +0.750 |
| Effective length of Bitterlick Stick | 26.218 |

\[
\text{Effective length} \times \frac{\text{Width of gauge}}{\text{per 1" of stick}} = \text{Gauge width}
\]

| 10 FACTOR WIDTH | 26.218 \times 0.0303 | = 0.7944" |
| 5 FACTOR WIDTH  | 26.218 \times 0.0214 | = 0.5611" |

**Figure 4. Construction of angle gauges.**

c. The Panama Angle-Gauge is also a horizontal angle-gauge. This instrument is a short tube with a peephole on one end and a modified cross-arm on the other (Fig. 5).

d. An angle-gauge can be improvised using coins, thumbnails, sticks, etc., when needed in an emergency. However, one must know the correct width of the substitute cross-arm and the reach in order to determine the basal area factor. (See Table 4 for formulae.)

e. A wedge prism is a wedge-shaped piece of optically ground glass.
FIGURE 5. Panama angle-gauge.

The principle of the wedge prism as used in point-sampling is based upon the deflection or displacement of light rays. Prism strengths are measured in diopters. A prism of 1 diopter will displace an object 1 unit in 100 units. A 3-diopter prism therefore displaces an object 3 feet in 100 feet.

Three-diopter prisms are commonly used because they have a basal area factor very close to 10 and are reasonable in cost. If ground exactly, 3-diopter prisms would establish a ratio of 1:33\(\frac{1}{3}\) rather than 1:33 and have a basal area factor of 9.8 rather than 10. For a prism to have an exact basal area factor of 10 the prism must have a diopter strength of 3.03. It is possible when purchasing a number of inexpensive 3-diopter prisms to find some having a diopter strength of 3.03.

A wedge prism with an exact basal area factor of 10 has an angle of deflection that is equal to the angle of incidence of an angle-gauge with the same basal area factor. This angle is commonly referred to as the critical angle (Fig. 6).

**Theory of Point-Sampling**

Anyone being initiated to this new method of determining basal area without measuring tree diameters or plot distances may be puzzled as to how it

FIGURE 6. Principle of wedge prism. (Object A appears as if at A'.)
works. Several explanations of the theory are offered here to help in understanding the point-sampling concept. A firm grasp of the theory, however, can only be had by working on the ground with the instruments and also by acquiring a thorough understanding of the formulae and tables.

Following are three explanations of the theory based upon three different approaches. All following explanations and sketches apply to an angle of 104.18 minutes which gives the resultant basal area factor of 10. The principles of the theory, however, apply to any angle and its respective basal area factor.

Explanation No. 1

The angle-gauge or prism establishes a ratio between the diameter of a tree that is just on the edge of the "plot"* (and is counted) and the distance from the center of the tree to the point center. With a basal area factor of 10 as mentioned above, the ratio is 1:33. This ratio exists for all tree diameters and is expressed as the fraction, diameter of tree in feet over distance from center of tree to point center in feet (Fig. 3).

A ratio also exists between the area in square feet of a tree that is just in and the area of the circle whose radius is the distance from the tree to the point center. The area of the large circle is 4356 times the area of the tree and this holds for all tree diameters and their respective "plot" areas. Since there are 43,560 square feet in an acre, each counted tree therefore represents 1/4356 of an acre or 10 square feet. Recall that the areas of circles vary with the squares of their diameters. As an example: Assume a tree 1 foot in diameter and just in at 33 feet. Then their respective areas vary as \( d^2 \) to \( D^2 \). Therefore:

\[
\frac{d^2}{D^2} \quad \frac{\text{tree diam.}}{\text{plot diam.}} = \frac{1^2}{66^2} = \frac{1}{4,356}
\]

Explanation No. 2

Assume that all trees in the forest are encircled with imaginary rings or zones whose diameters are 66 times the diameter of the respective trees (Groshenbaugh, 1955). Naturally the larger the tree, the larger this ring. In point-sampling, the point center would have to fall within these rings in order for such trees to be counted (Fig. 7). The angle-gauge or prism determines if the point center falls within these rings.

The probability of this point center falling within the ring is proportional to the size of the tree. A tree twice the diameter of another would have four times the probability of being counted as would the smaller. This provides a good sampling method where larger trees which are usually more variable in volume and quality are sampled in greater proportion than the smaller trees which are less variable and of less importance. It is a drastic change.

* "Plot" or "plot" area means the area represented by each tree diameter and does not refer to a fixed acreage.
from conventional methods where trees are sampled in proportion to their distribution in the stand.

Each counted tree, while representing 10 square feet of basal area per acre, also represents a certain part of an acre and a certain number of trees per acre. A small tree will represent a small area but a large number of trees. For example, an 8-inch tree with a basal area of .349 square feet is on a "plot" with an area of .034889 acres. It takes 28.65 "plots" of this size to make an acre. Each 8-inch tree therefore represents 28.65 trees on an acre; 28.65 \times .349 equals 10 square feet of basal area. At the other extreme, a 42-inch tree represents a "plot" of approximately one acre. A tree this size has about 10 square feet of basal area, and, 10 \times 1 equals 10. Again it should be pointed out that each counted tree irrespective of its size represents 10 square feet of basal area per acre.

**Explanation No. 3**

Further explanation is offered from a mathematical standpoint (Afanasie, 1957). The basal area of a tree \( \pi r^2 = \frac{\pi D^2}{4} = .785D^2 \) square inches or \( \frac{.785D^2}{144} = .00545D^2 \) square feet. The size of the "plot" within which trees with the Diameter D are counted \( \pi r^2 = .R \). R of the "plot" is 33 times as large as D, hence:
Plot area (sq. in.) = \pi \times (33 \times D)^2

Plot area (sq. ft.) = \frac{\pi \times (33 \times D)^2}{144} = 23.7042D^2

Plot area (acres) = \frac{23.7042D^2}{43,560} = .000545D^2

From the above it is evident that the ratio of the basal area of one tree to the size "plot" (acres) is 10 to 1 (.000545D^2/.000545D^2). This relationship holds for a tree of any size.

The basal area per acre represented by one counted tree would be:

\[
\frac{1 \text{ Acre}}{\text{Plot Area}} = \frac{\text{basal area per acre}}{\text{basal area of tree}}
\]

\[
\frac{1}{.000545D^2} = \frac{\text{b.a. per acre}}{.00545D^2}
\]

b.a. per acre = \frac{1 \times .00545D^2}{.000545D^2} = 10

A counted tree regardless of its size or position within the "plot" represents 10 square feet of basal area per acre when using an instrument with a basal area factor of 10. In addition, each counted tree regardless of its position within the "plot" represents a given number of trees per acre and a certain part of an acre depending upon its size. In all cases the actual basal area of a counted tree when multiplied by the number of trees it represents per acre will always equal 10 (Table 5).

A cruiser taking a point in large timber where all trees are well within the limits of the "plot" may think that the basal area is more than he reads with his angle-gauge or prism. It must be remembered, however, that the gauge or prism is giving a per acre figure and not a figure for the area around the point. For this reason no one should ever attempt to compare a point with a fixed plot. A point has no area—trees on the "plot", however, represent certain areas. For example: assume a point with 10 large pine trees each about 52 inches in diameter. The gauge gives a reading of 100 square feet yet the 10 trees actually total about 150 square feet in basal area. But, a 52-inch tree represents a "plot" or zone area of 1.5 acres, therefore reducing the basal area to an acre basis, or \[
\frac{150}{1.5} = 100 \text{ square feet per acre (Fig. 8)}.
\]

**Correct Use of Instruments**

**Angle-Gauge**

With the eye as the point center, the cruiser counts all trees whose diameters at breast height appear larger than the crossarm. Where the trees appear the same size as the crossarm one can either count every other tree or measure the distance to the tree, measure its diameter and multiply the
diameter (inches) by a plot radius factor (2.75 for a 10 BAF). If the product is larger than the distance, the tree is counted. Each counted tree is multiplied by the basal area factor to give the basal area per acre. With trees that lean to the right or left of the line of sight, turn the angle-gauge until the crossarm is at right angles to the stem. Trees that lean toward or away from the observer can generally be handled like normal trees.

Trees that are forked above breast height should be counted as one tree for basal area. Trees forked below breast height should be counted as two trees. Be sure to maintain the eye on the point center when making the tree count. It is important to hold the gauge the same distance from the eye as allowed for in determining length of stick. Errors of up to 5 percent or more can result from improper positioning of the gauge.

One must be certain of tallying all trees and especially those larger trees that may be hidden at some distance from the center. Since the cruiser counts trees from a fixed spot, he must make sure not to count dead ones or miss any
that may be hidden by other trees. Care must be exercised to maintain the
same distance from a hidden tree to point center when necessary to move
off point center to view the hidden tree. Where only merchantable trees are
to be tallied, one can select a small sapling as a pivot point to be certain of
maintaining the point center. However, where the total basal area is to be
tallied, selecting a small sapling as plot center should be avoided as this tree
will automatically give 10 square feet (BAF 10) whereas under ordinary
circumstances few trees this size may actually be counted.

In hilly terrain allowances must be made for slopes that exceed 15 per-
cent (Table 6). Although slope correction tables are available, actually a
separate correction factor would have to be applied to almost every tree on
the "plot". The only cases where constant slope correction factors for all
trees can be applied would be at the bottom of a perfect bowl or the top of
a perfect knob (cone). Therefore, in hilly country the Spiegel-Relaskop
would appear to be the best instrument to use since it automatically corrects
for slope.

Prism

The prism is held at any convenient distance from the eye, but the center
of the prism is the point center rather than the eye since the angle begins at
the prism. In practice a spot directly beneath the prism becomes the plot
center. The observer counts all trees whose stem sections at breast height
when viewed through the prism do not appear to be detached from the
main stem. Borderline trees are handled similarly to those described in the
angle-gauge discussion. Likewise, each counted tree is multiplied by the
basal area factor to give the basal area per acre.

Great errors can arise from the improper positioning of the prism. Correct
use requires that it be held in a vertical position and at right angles to the
line of sight (Fig. 9a). On level ground the top edge should be horizontal.
With the top edge held horizontal, rotate prism slightly to determine posi-
tion of minimum deflection. At this position it can be assumed that the prism
is being held at right angles to the line of sight.

A horizontal rotation and/or a dipping of the prism toward or away from
the observer increases the dioptrre strength because of the difference in glass
thickness (Fig. 9b). Any rotation within a vertical plane decreases the
dioptrre strength (Fig. 9c). Deviations of 5 degrees in positioning the prism
will cause no measurable errors, but greater deviations can introduce very
significant errors. Extreme carelessness in positioning the prism can change
the resultant strength of a 3-dioptrre prism by nearly 1 dioptrre.

Other precautions used with the gauge such as hidden trees, forked trees
and maintaining the point center also apply. In viewing trees that lean to
right or left, tip the prism so that the top is perpendicular to the bole of the
tree. Correction for slope can be made by tipping the top edge to the same
angle as the slope. In this manner the prism automatically corrects for slope
distance (Fig. 10). For this reason and the ease of carrying, the prism
has assumed considerable popularity. Careful use of the prism in dense woods is necessary because of the difficulty of maintaining individual tree identity. Most persons, however, will find that the angle-gauge is probably more reliable since there is less chance to err. Various devices have been made to hold the prism level, perpendicular and at right angles to the observer's eye so as to cut down human error. These devices have been frequently described in the Journal of Forestry and various other forestry publications.
Practical Tips on Use of Instruments

Through field use a number of handy tips have evolved. A few of these are:

1. With borderline trees, use a Bitterlich tape. Measure the tree diameters to correlate this with the Bitterlich tape measurement. If the tree diameters are equal to or larger than the readings on the Bitterlich tape, the trees must be counted—if diameters are smaller, then the trees are not counted. A Bitterlich tape can be easily made by printing full inch diameters at 2.75 foot (BAF 10) intervals on the reverse side of a cloth tape.

2. In measuring trees to determine if they are “in” or “out,” they should be calipered perpendicular to the line of sight.

3. Always attempt to start the tree count from the same cardinal direction so as to be consistent.

4. Make a double-check of “plot” before leaving it.

5. Where possible, use devices such as a Purdue point-sampling block for holding prisms steady.
6. Darken the crossarm on an angle-gauge for better contrast.
7. Mark the basal area factor on the crossarm where using two or more different basal area factors.
8. Consider a penny as an emergency gauge. The “crossarm” is \( \frac{3}{4} \) inch, and if held at \( 24\frac{3}{4} \) inches, the penny gives a basal area factor of 10.
9. When sighting with the angle-gauge, line one side of the crossarm with one side of a tree. If bark appears on the other side of the crossarm, the tree should be counted.
10. Rest the gauge on some object such as a Jacob staff to obtain greater accuracy.
11. Adjust eye shade properly in order to obtain clearer readings with the Spiegel–Relaskop.

Determining the Basal Area Factor

If an angle-gauge is carefully constructed, there should be no need for checking it for the exact basal area factor. It would be a good idea, however, to do so as an added precaution.

Prisms, and particularly the inexpensive ones, may not be ground to the exact diopter desired. The more expensive prisms are guaranteed to have a maximum tolerance of one minute from the specified angle of 104.18 minutes required for a 3.03-diopter prism with a BAF of 10. This means a 3.03-diopter glass could have a BAF ranging from 9.8 to 10.2 and a possible error in the field of \( \pm 2 \) percent (Bower et al., 1959).

Greater variations of course will be found in the inexpensive prisms where the diopter strength may vary by 0.1 diopter more or less. Because of this, one cannot assume that a 3-diopter prism has a BAF of 9.8 and a 2-diopter one of 4.356. An inexpensive prism could be off 10 percent or more even before human errors are introduced. However, it might be desirable and cheaper in the long run to buy a number of inexpensive prisms and select for use only the more exact ones.

All prisms should be calibrated to determine the exact basal area factor and this figure etched on the glass or scratched on the pouch. Several calibrations should be made for each prism to arrive at the exact factor. Even with a prism ground to the exact diopter or BAF, it is necessary for each individual using that prism to calibrate it since eyesights vary. The calibration is done as follows:

1. Place a rectangular target of exactly 1 foot (or any carefully measured width) vertically against a contrasting background. Do not use a tree for this calibration unless its diameter has been calipered
2. Back away from the target, and viewing the target at eye level through the prism, exactly line up the right side of displaced portion with the left side of the target, with prism held in right hand and thin edge to left (Fig. 11).

20
3. Measure distance in feet from target to prism.
4. Using formula below, compute BAF.

$$BAF = 10,890 \left( \frac{w}{d} \right)^2$$

where \( w \) = width of target in feet
\( d \) = distance from target to prism in feet

The same procedure and the same formula can be used to calibrate an angle-gauge. With an angle-gauge the edges of the crossarm must line up with the edges of the target.

Without any magnification of the instruments, there can be significant differences in the calibration of basal area factors between individuals. The main reason for this, of course, is in making decisions on borderline trees. Therefore, with gauges having no magnification it would seem possible that calibrated basal area factors can be rounded slightly for field use—for example, 9.9 to 10.

When the basal area factor has been determined for a particular gauge or prism, each counted tree will represent a number of square feet equal to the calibrated basal area factor. Where an odd BAF is encountered, a handy reference table should be prepared showing various tree number and basal area relationships to simplify field computations.
Once basal area factors have been determined, the relationship of volume between them is directly proportional, and the relationship of area is inversely proportional. Up to this point computations and relationships are largely based on squares or square roots. Recall how areas of circles vary with the squares of their diameters.

**USE OF BASAL AREA AND POINT-SAMPLING IN FOREST MANAGEMENT**

**As a Measure of Stocking**

Basal area has come into the limelight in recent years as a measure of stocking levels and consequently as a basis for cutting recommendations and marking guides. In the Lake States, tables of recommended levels of stocking using basal area per acre have been prepared for most timber types. The use of basal area as a level of stocking represents a convenient and unbiased method of determining the intensity of a thinning or harvest cut.

As an example, the following is quoted from Arbogast (1953) and refers to stocking in northern hardwoods:

"When we started using the selection system we had to talk the language of the logger and mill owner in order to sell the idea that they could afford to cut selectively. It was natural, therefore, that we turned to a unit of measurement with which they were familiar, net board feet per acre, to express stocking level. We soon realized that this would not work. Volume is an expression of site. So if we were going to use volume as a stocking guide, we would also have to include some sort of site index, as the stands varied from 4,000 to 15,000 net board feet per acre. We had no precise expression of such site indexes.

"The next attempt to express stocking was in terms of the percent of the total net merchantable stand to be cut. For example, we spoke of '30 percent cuts' and '50 percent cuts'. It was an improvement because now site was eliminated. Although this method is still used extensively, it has many disadvantages. Some of them are:

(1) It does not take into consideration the condition of the timber. A stand with much cull is actually cut heavier at a given percent than one with little cull. Using gross volume as a base would eliminate this difficulty.

(2) Net board feet measurements are very difficult to make in the forest, because only indirect methods can be used. No matter how carefully the basic measurements are made, large errors are possible because of the nature of the unit.